The Subjective Risk and Return Expectations of Institutional Investors

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Abstract

We use the long-term Capital Market Assumptions of major asset managers and institutional investor consultants from 1987 to 2022 to provide three stylized facts about their subjective risk and return expectations on 19 asset classes. First, there is a strong and positive subjective risk-return tradeoff, with most of the variability in subjective expected returns due to variability in subjective risk premia (compensation for market beta) as opposed to subjective alphas. Second, belief variation and the positive risk-return tradeoff are both stronger across asset classes than across institutions. And third, the subjective expected returns of these institutions predict subsequent realized returns across asset classes and over time. Taken together, our findings imply that models with subjective beliefs should reflect a risk-return tradeoff. Additionally, accounting for this subjective risk-return tradeoff when modeling multiple asset classes is even more important than incorporating average belief distortions or belief heterogeneity in our setting.

JEL Classification: G11, G12, G23

Keywords: Institutional Investors, Subjective Beliefs, Subjective Expected Returns, Subjective Risk, Subjective Risk Premia.

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Introduction

The risk-return tradeoff in financial markets is at the heart of asset pricing. The literature typically explores this topic by linking risk measures of financial assets to their subsequent realized returns (e.g., Fama and MacBeth (1973)). This approach recovers the risk-return tradeoff investors perceive if they have rational expectations (i.e., their expectations are objective in that they reflect the true data generating process in the economy). However, if investors' subjective risk and return expectations deviate from rational expectations, then the risk-return relation in the realized return data is not very informative about the risk-return tradeoff investors perceive.

The existing literature on subjective beliefs (reviewed by Adam and Nagel (2023)) documents important deviations between the objective and subjective expected returns of different economic agents, with large asset managers having subjective equity premia that better reflect the cyclical properties of the objective equity premium (Dahlquist and Ibert (2023)). However, there is little work on the connection between subjective risk and return expectations, particularly for institutional investors and across asset classes. As Adam and Nagel (2023) put it, "*We need more work that explores how investors...risk perceptions are linked to the subjective risk premia that they demand to hold risky assets.*"

In this paper, we fill this important gap in the literature. Specifically, we study the long-term Capital Market Assumptions (CMAs) of major asset managers and institutional investor consultants reflecting their subjective risk and return expectations on 19 asset classes from 1987 to 2022. We uncover three stylized facts. First, as Figure 1(a) shows, there is a strong and positive subjective risk-return tradeoff. In particular, most of the variability in subjective expected returns is driven by variability in subjective risk premia (compensation for market beta) as opposed to subjective alphas. Second, we find that belief variation and the positive risk-return tradeoff are both stronger across asset classes than across institutions. This implies the positive relation between subjective risk and return expectations is more influential than belief disagreement in our setting. And third, the subjective beliefs of these institutions

(a) Subjective $\mathbb{E}[\boldsymbol{Risk}]$ vs $\mathbb{E}[\boldsymbol{Return}]$ **(b) $\mathbb{E}[\boldsymbol{Return}]$: Beliefs vs Data**

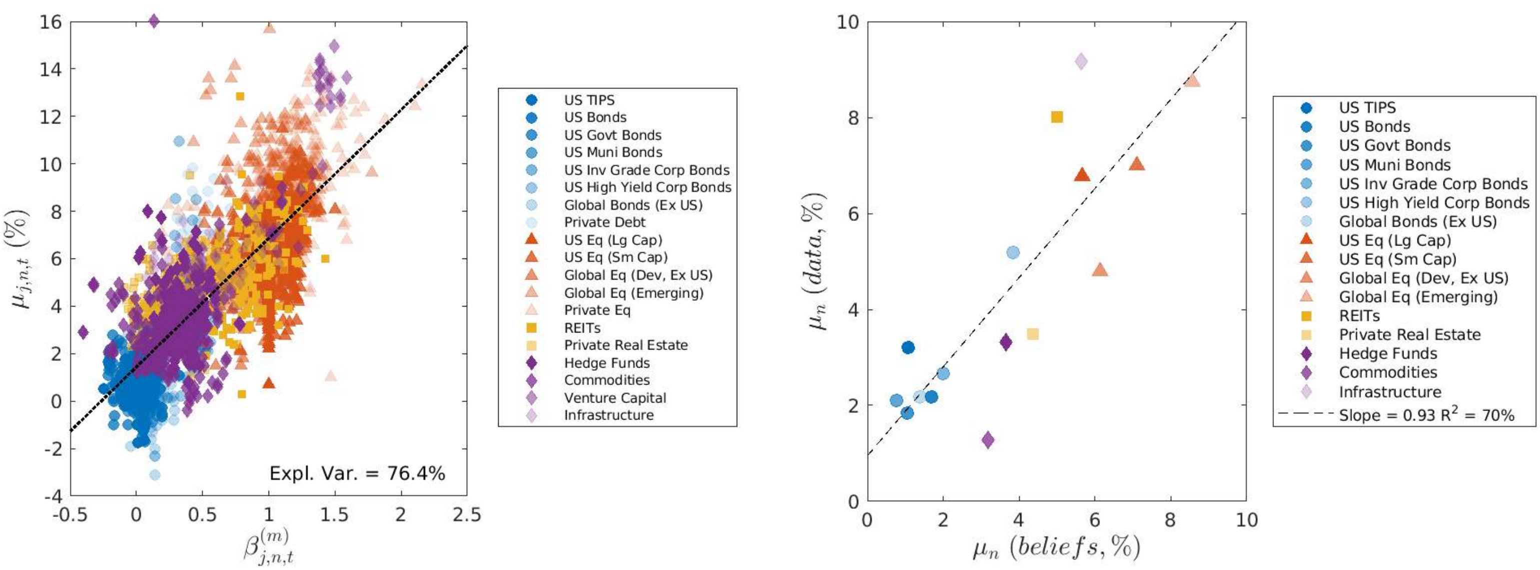

Figure 1

Summary of Main Results

Panel (a) plots subjective expected excess returns ($\mu_{j,n,t}$) against subjective market betas ($\beta^{(m)}_{j,n,t}$), with the market reflecting US equities. These quantities are obtained from our CMAs and vary across investors (j) and asset classes (n) as well as over time (t). Panel (b) plots subjective μ_n (aggregated over time and across investors) against the respective μ_n obtained from average realized excess returns. See Section 2 for more details about our subjective beliefs data and Section 4 for the matching of beliefs to realized returns.

are largely (but not fully) consistent with objective beliefs. For instance, Figure 1(b) shows that a regression of average realized returns on average subjective expected returns leads to a slope coefficient close to 1 and an R^2 around 70%, with similar results for data moments related to expected risk (i.e., volatilities and betas). Moreover, the subjective beliefs of these institutions predict subsequent realized returns across asset classes and over time, but they do not predict subsequent risk or alphas in the time series. Taken together, our findings imply that models with subjective beliefs should reflect a risk-return tradeoff. Additionally, in our setting, accounting for this risk-return trade-off when modeling subjective beliefs across asset classes is even more important than incorporating average belief distortions or belief heterogeneity (despite the presence of both in the subjective beliefs data to some degree).

We start by formalizing the point that the link between an asset's objective risks and its subsequent realized returns is only informative about the risk-return tradeoff perceived by investors if investors have rational beliefs on average. In particular, we show that if investors are optimistic (i.e., overweight probabilities of good outcomes), then empirical tests based on realized returns may "reject" a valid asset pricing model. In contrast, if investors are pessimistic (i.e., overweight probabilities of bad outcomes), then empirical tests based on realized returns may "accept" an invalid asset pricing model. So, to generally test the validity of a hypothesized subjective risk-return tradeoff, one needs to use data on the beliefs of investors.

In turn, we describe our data on the CMAs of asset managers and institutional investor consultants, which cover cash as well as the 19 asset classes listed in the legend of Figure 1(a). For each institution-year observation, our CMAs data provide expected returns as well as expected volatilities and expected return correlations across asset classes. We use these data to create the excess of cash counterpart for each of these belief quantities. We then construct subjective expected excess returns ($\mu_{j,n,t}$) and subjective CAPM market betas ($\beta^{(m)}_{j,n,t}$) for each institution (j), asset class (n), and year (t). We use two market proxies for the betas, one based on the US equity large cap asset class (referring to the underlying model as the "Equity CAPM") and another based on the aggregate portfolio of US pension funds (referring to the underlying model as the "Pension CAPM"). Combining these market betas with the subjective expected return on the respective market proxy ($\mu^{(m)}_{j,t}$) allows us to calculate subjective risk premia, $rp^{(m)}_{j,n,t} = \beta^{(m)}_{j,n,t} \cdot \mu^{(m)}_{j,t}$, and subjective alphas, $\alpha^{(m)}_{j,n,t} = \mu_{j,n,t} - rp^{(m)}_{j,n,t}$.

We then study the subjective risk-return tradeoff and decompose $\mu_{j,n,t}$ variation into variation in $rp^{(m)}_{j,n,t}$ and $\alpha^{(m)}_{j,n,t}$. Regardless of the market proxy we use, we find that deviations of $\mu^{(m)}_{j,t}$ from the market risk premia (estimated from regressions of μs on βs) are economically small. We also find that the average subjective alpha of each asset class is economically small. Moreover, we show that most of the variability in $\mu_{j,n,t}$ comes from variation in $rp^{(m)}_{j,n,t}$ and not $\alpha^{(m)}_{j,n,t}$. Specifically, we find that more than 75% (90%) of the variability in $\mu_{j,n,t}$ is driven by $rp^{(m)}_{j,n,t}$ in the Equity CAPM (Pension CAPM), with less than 25% (10%) driven by $\alpha^{(m)}_{j,n,t}$.

While alphas play a relatively small role in explaining the overall variation in expected returns, they play a large role in explaining disagreement across institutions. Specifically, when we focus on $\mu_{j,n,t}$ variation across institutions (e.g., by adding fixed effects for n and t), then alphas explain around 50% (40%) of the variability in subjective expected returns in the Equity CAPM (Pension CAPM). As such, our results indicate that there is considerable disagreement across institutions about alphas within asset classes, but this disagreement is small relative to the difference in risk compensation they expect across asset classes.

The prior paragraph highlights the fact that belief heterogeneity across institutions (i.e., disagreement) and belief heterogeneity across asset classes (i.e., cross-asset risk-return tradeoff) play distinct roles in shaping overall beliefs. To further explore the different sources of belief heterogeneity, we decompose within year variability in expected returns through institution fixed effects and asset class fixed effects. We show that asset class fixed effects explain more than 80% of the within year variability in subjective expected returns in the typical year. This result is striking and explains why expected return variation is largely driven by subjective risk premia and not subjective alphas. Specifically, while alphas are important in explaining expected return variation across institutions within a given asset class, the variation in expected returns across institutions, though non-trivial, is overwhelmed by the large variation in expected returns across asset classes. We show that a similar result holds when we model belief heterogeneity across institutions in a different way that allows for institutions to have different forms of heterogeneity for different asset classes.

Having established a positive and strong subjective risk-return tradeoff, we turn to exploring average belief distortions. We find that belief distortions only play a relatively modest role in our setting. For example, average subjective expected returns, volatilities, and betas all line up well with their respective realized return counterparts. Specifically, regressions across asset classes of expected return moments on their realized return counterparts yield slope coefficients that are close to 1 and have high R^2s. In contrast, there is effectively no connection between subjective alphas and subsequent realized alphas across asset classes.

The link between subjective beliefs and realized return moments is not only present across asset classes on average, but also when we compare subjective beliefs with next-year realized return moments. For instance, a pooled panel regression of annual realized returns on the subjective expected returns formed at the end of the prior year (aggregated across institutions) has an intercept close to zero and a slope coefficient close to one (both economically and statistically), with a predictive R^2 above 7%. Remarkably, out-of-sample R^2 values (R^2_{OOS}) that contrast subjective expected returns with historical average returns are even higher. In particular, $R^2_{OOS} = 12.6\%$ if we focus on predictability across assets and $R^2_{OOS} = 12.9\%$ if we focus on predictability over time. Similar results (with stronger R^2 values) are observed in analogous pooled panel regressions of 1-year realized risk measures on subjective expected risk. One stark difference, however, is that subjective expected risk does not predict subsequent realized risk in the time series. Alphas provide another source for belief distortions since subjective alphas do not predict subsequent realized alphas across asset classes or over time. In summary, subjective beliefs largely predict subsequent realized return moments, but we also identify a few mismatches that lead to belief distortions. As such, there is scope to further improve the belief formation process of these institutions.

Contribution to the Literature

We provide four contributions to the subjective beliefs literature in asset pricing (summarized by Adam and Nagel (2023)).[1] First, we investigate the connection between subjective risk and return expectations across asset classes instead of studying the cyclical properties of subjective expected returns as it is common in this literature. Second, we explore beliefs

[1] Some papers in this literature are Froot (1989), Bondt (1993), Vissing-Jørgensen (2003), Bacchetta, Mertens, and Wincoop (2009), Brav, Lehavy, and Michaely (2005), Bacchetta, Mertens, and Wincoop (2009), Malmendier and Nagel (2011, 2016), Chen, Da, and Zhao (2013), Amromin and Sharpe (2014), Greenwood and Shleifer (2014), Piazzesi, Salomao, and Schneider (2015), Adam, Marcet, and Bautel (2017), Cassella and Gulen (2018), Cieslak (2018), Wu (2018), Bordalo et al. (2019), Adam, Matveev, and Nagel (2021), De La O and Myers (2021), Giglio et al. (2021) Malmendier, Nagel, and Yan (2021), Nagel and Xu (2021, 2022), Wang (2021), Andonov and Rauh (2022), Bastianello (2022), Beutel and Weber (2022), Bordalo et al. (2022), Cassella et al. (2022), Chaudhry (2022), Dahlquist and Ibert (2023), De La O, Han, and Myers (2022), Gandhi, Gormsen, and Lazarus (2022), Gormsen and Huber (2022, 2023), Jensen (2022), Jo, Lin, and You (2022), Li (2022), Lochstoer and Muir (2022), Loudis (2022), Atmaz et al. (2023), Boons, Ottonello, and Valkanov (2023), Dahlquist, Ibert, and Wilke (2023), Gnan and Schleritzko (2023), Egan, MacKay, and Yang (2023), Laudenbach et al. (2023), and Begenau, Liang, and Siriwardane (2023).

for many asset classes instead of focusing on a single asset class as most of the literature does (Bacchetta, Mertens, and Wincoop (2009), Andonov and Rauh (2022), Jo, Lin, and You (2022), and Nagel and Xu (2022) are notable exceptions). Third, we study the beliefs of institutional investors and their consultants instead of the beliefs of households, individual investors, professional forecasters, and sell-side financial analysts as most of the literature does (Andonov and Rauh (2022) and Dahlquist and Ibert (2023) are notable exceptions). And fourth, we demonstrate that subjective beliefs and realized returns have a tight link across asset classes and over time, in stark contrast to prior findings based on other agents.

The closest paper to ours is Dahlquist and Ibert (2023). They explore the CMAs of asset managers (with a brief analysis of consultants in their appendix) and find that their subjective equity premia are countercyclical, with the degree of countercyclicality roughly matching that of the objective equity premium. They also study the pass through of the subjective equity premia to portfolio allocations, finding a degree of pass through that is stronger for institutions than it is for individual investors (but weaker than in frictionless one-period models with rational expectations).

We add to Dahlquist and Ibert (2023) in four important ways. First, we study 19 risky asset classes simultaneously instead of exploring only equities. Second, we focus on the link between subjective risk and return expectations across asset classes, which they do not study. Third, we show that subjective risk and return expectations vary much more across asset classes than across institutions, an entirely novel result relative to the prior literature. And fourth, we provide a comprehensive analysis of the link between subjective risk and return expectations with their respective realized return moments.

It is also important to point out that we expand the Dahlquist and Ibert (2023) sample in the time-series dimension. Specifically, while the sample from Dahlquist and Ibert (2023) is sparse before 2010, we have reasonable coverage in the 2000s and even some (modest) coverage in the late 1990s (plus one consultant from 1987 to 1996). We accomplish this time series coverage through a data collection process that relies heavily on direct interactions with the institutions themselves (with online searches and direct contacts with pension funds used as complementary methods). Using our longer time series, we affirm and extend one of the main findings in Dahlquist and Ibert (2023), namely that the subjective equity premium is countercyclical (i.e., it moves together with the S&P 500 earnings yield).

Finally, a relatively small subset of the subjective beliefs literature explores the connection between risk and subjective expected returns. We can broadly categorize these papers into three groups. The first group relies on subjective risk and return measures from surveys of households or individual investors and finds a negative relation between expected returns and subjective risk (e.g., Amromin and Sharpe (2014), Giglio et al. (2021), Jo, Lin, and You (2022), and Gnan and Schleritzko (2023)).[2] The second group links the subjective expected returns of financial professionals (e.g., financial analysts and CFOs) to subjective or objective measures of risk (e.g., betas from factor models) in the cross-section of stocks and finds a strong link between expected returns and subjective or objective risk (e.g., Brav, Lehavy, and Michaely (2005), Wu (2018), Bastianello (2022), Jensen (2022), and Gormsen and Huber (2023)). The third group identifies a positive time-series link between subjective expected returns and measures of objective and subjective volatility from individual investors and financial analysts (Wu (2018), Bastianello (2022), and Nagel and Xu (2022)).

We add to these papers by studying the subjective risk-return tradeoff across asset classes under the subjective measure of asset managers and institutional investor consultants (who have a strong potential to affect asset prices given their direct or indirect influence on large portfolio holdings). The fact that we observe subjective betas with respect to particular risk factors and simultaneously observe the factors' subjective expected returns also allows us to decompose the subjective expected return variation across asset classes into the effect of subjective risk premia and alphas, which is entirely new in the literature.

We next provide theoretical motivation to guide our empirical analysis of subjective beliefs.

[2] Interestingly, Gnan and Schleritzko (2023) find a positive correlation between subjective volatility and expected returns for the aggregate market after controlling for whether the individual investor perceives the overall market as overvalued/undervalued. This result suggests that individual investors require a positive risk compensation despite perceiving a negative risk-return tradeoff in equilibrium.

1 Theoretical Motivation

In this section, we suppress the time index, t, to simplify exposition. However, our analysis in this section can be thought of as reflecting time t beliefs about time $t+h$ outcomes for an arbitrary h.

Suppose the Stochastic Discount Factor (SDF) for a set of $j = 1, 2, ..., J$ investors is given by M. If these investors have access to $n = 1, 2, , ..., N$ assets (or asset classes) over time, then we have

$$1 = \mathbb{E}_j[M \cdot R_n] \qquad \Rightarrow \qquad \mathbb{E}_j[r_n] = \beta_{j,n} \cdot \lambda_j \tag{1}$$

where $r_n = R_n - R_f$, $\beta_{j,n} = \frac{\mathbb{C}ov_j[-M, r_n]}{\mathbb{V}ar_j[M]}$, and $\lambda_j = \frac{\mathbb{V}ar_j[M]}{\mathbb{E}_j[M]} > 0$.

Equation 1 shows that investor j perceives a positive risk-return tradeoff. That is, investor j believes that assets that negatively comove with M deliver high expected returns. In typical models, M reflects marginal utility so that risky assets are the ones that investors think will perform poorly in states of nature where they have high marginal utility.

It is important to note that $\mathbb{E}_j[\cdot]$ represents investor j subjective expectation operator, which can differ from the objective expectation operator, $\mathbb{E}_o[\cdot]$. As such, assets that investor j perceives to have high beta may not actually have high beta. Similarly, assets that investor j perceives to have high expected return may not actually have high expected return. Taken in isolation, the only theoretical restriction Equation 1 provides is that subjective expected returns are positively linked to subjective expected risk (i.e., subjective betas).

From Equation 1, we can recover the objective risk-return tradeoff. To start, let $\pi_{j,s}$ reflect investor j subjective probability that state of nature s will materialize next period. Then, aggregating Equation 1 across investors yields

$$1 = \frac{1}{J} \cdot \sum_{j=1}^{J} \mathbb{E}_j[M \cdot R_n] = \frac{1}{J} \cdot \sum_{j=1}^{J} \sum_{s=1}^{S} \pi_{j,s} \cdot M_s \cdot R_{n,s} = \sum_{s=1}^{S} (M_s \cdot R_{n,s} \cdot \underbrace{\frac{1}{J} \cdot \sum_{j=1}^{J} \pi_{j,s}}_{\pi_s}) \tag{2}$$

which can be written as

$$1 = \sum_{s=1}^{S} \pi_s \cdot M_s \cdot R_{n,s} = \sum_{s=1}^{S} \pi_{o,s} \cdot \Pi_s \cdot M_s \cdot R_{n,s} \; = \; \mathbb{E}_o[\Pi \cdot M \cdot R_n] \tag{3}$$

where $\Pi_s = \pi_s/\pi_{o,s}$ reflects the average probability distortion for state of nature n (i.e., "investor sentiment"). As such, the distorted SDF, $\Pi \cdot M$, captures the risk-return tradeoff in the realized return data (i.e., under the objective measure):

$$1 = \mathbb{E}_o[\Pi \cdot M \cdot R_n] \qquad \Rightarrow \qquad \mathbb{E}_o[r_n] = \beta_{o,n} \cdot \lambda_o, \tag{4}$$

where $\beta_{o,n} = \frac{\mathbb{C}ov_o[-\Pi \cdot M, r_n]}{\mathbb{V}ar_o[\Pi \cdot M]}$, and $\lambda_o = \frac{\mathbb{V}ar_o[\Pi \cdot M]}{\mathbb{E}_o[\Pi \cdot M]} > 0$

Since we rarely observe the beliefs of investors, we typically assume investors have rational/objective beliefs on average (i.e., $\Pi_s = 1$). As such, we often test an SDF model ($\widehat{M}$) using the moment condition $\mathbb{E}_o[\widehat{M} \cdot R_n] = 1$. Such a test can lead to two important mistakes:

1. We may "reject" a valid SDF. Suppose $\widehat{M} = M$ so that our SDF model is valid. If Π comoves positively with $1/M$, then the risk-return tradeoff based on $\widehat{M}$ will look weaker than investors perceive it to be. For instance (at the extreme), if $\Pi \propto 1/M$ then there is no risk-return tradeoff under the objective measure even though investors perceive a positive risk-return tradeoff. We will conclude that exposure to $\widehat{M}$ is not priced by investors even though it is. This extreme scenario could arise if beliefs are distorted optimistically, for instance. Specifically, $\Pi_s = \pi_s/\pi_{o,s} \propto 1/M_s$ implies investors assign relatively high probabilities to states of nature with low M (good states with low marginal utility) and relatively low probabilities to states of nature with high M (bad states with high marginal utility). $\mathbb{C}or_o(\Pi, 1/M) > 0$ leads to a similar qualitative result.

2. We may "accept" an invalid SDF. Suppose $\widehat{M} \neq M$ so that our SDF model is invalid. If Π comoves positively with $\widehat{M}/M$, then the risk-return tradeoff based on $\widehat{M}$ will look more positive than investors perceive it to be. For instance (at the extreme), if $M = 1$ and $\Pi \propto \widehat{M}$ then there is a positive risk-return tradeoff under the objective measure even though investors perceive no risk-return tradeoff. We will conclude that exposure

to $\widehat{M}$ is priced by investors even though it is not. Such an extreme scenario could arise if investors are risk-neutral but beliefs are distorted pessimistically. Specifically, $\Pi_s = \pi_s/\pi_{o,s} = \widehat{M}_s$ implies investors assign relatively high probabilities to states of nature with high $\widehat{M}$ (states with high model-based marginal utility) and relatively low probabilities to states of nature with low $\widehat{M}$ (states with low model-based marginal utility). $\mathbb{C}or_o(\Pi, \widehat{M}/M) > 0$ leads to a similar qualitative result.

This analysis illustrates that it is important to understand the subjective risk-return tradeoff when there is concern that it may deviate from the objective risk-return tradeoff. Doing so helps to avoid incorrectly accepting or rejecting a particular model for the SDF. The rest of this paper quantifies the strength of the risk-return tradeoff based on the subjective beliefs of institutional investors about different asset classes. We focus on the Capital Asset Pricing Model (CAPM), so that periods of low market returns proxy for high marginal utility states (i.e., $\widehat{M} = a - b \cdot R_m$). Section 2 details our subjective beliefs data. Section 3 provides all results for the CAPM test under the subjective beliefs of institutional investors (i.e., we test $\mathbb{E}_j[\widehat{M} \cdot R_n] = 1$). Section 4 contrasts the subjective beliefs of institutional investors with realized returns (which reflect objective beliefs), thereby providing an initial (but not formal) analysis to determine whether Π can play an important role in the pricing of asset classes under the objective measure (i.e., in $\mathbb{E}_o[\Pi \cdot M \cdot R_n] = 1$).

2 The Subjective Beliefs Data

This section briefly describes our beliefs data. Subsection 2.1 introduces the concept of Capital Market Assumptions (CMAs), Subsection 2.2 describes our dataset of CMAs, and Subsection 2.3 provides a preliminary analysis to validate the data.

2.1 Capital Market Assumptions (CMAs)

Envestnet PMC states in their 2023 CMA Methodology report that

"Capital markets assumptions are the expected returns, standard deviations, and correlation estimates that represent the long-term risk/return forecasts for various asset classes. We use these values to score portfolio risk, assist advisors in portfolio construction, construct our own asset allocation models and create Monte Carlo simulation inputs for portfolio wealth forecasts."

Relatedly, JP Morgan states in their 2022 CMA report that

"We formulate our Long-Term Capital Market Assumptions (LTCMAs) as part of a deeply researched proprietary process that draws on quantitative and qualitative inputs as well as insights from experts across J.P. Morgan Asset Management. Our own multi-asset investment approach relies heavily on our LTCMAs: The assumptions form a critical foundation of our framework for designing, building and analyzing solutions aligned with our clients' specific investment needs."

As is clear from these statements, CMAs are important components of the underlying business for several large financial institutions. In fact, these institutions often have a team of highly trained financial experts dedicated to creating their CMAs, which reflect their long-term views on many asset classes (in our dataset, these long-term views have horizons ranging from 4 to 30 years, with 10 years being the most common horizon). In the case of institutional investor consultants, the CMAs are mainly used to advise their clients on portfolio allocation decisions. For instance, pension funds have portfolio allocation reports that rely heavily on the CMAs of their consultants, a feature we explore in our data collection process. In the case of asset managers (who often also have clients on their wealth management division), the CMAs are used both to advise clients and to guide the overall portfolio allocation of the different funds within the institution.

In contrast to the typical surveys about expected returns used in the literature, CMAs are not responses to questions designed by a third party research team. Instead, CMAs are fully developed documents produced organically by institutions. Moreover, CMAs tend to rely heavily on quantitative financial research, often containing citations to academic papers. As a consequence, it is plausible that CMAs reflect more sophisticated beliefs than the typical beliefs derived from surveys of individual investors or financial professionals. We provide evidence that largely supports this view.

2.2 Our Dataset of CMAs

Our dataset is based on beliefs extracted from the CMAs of asset managers (hereafter "managers") and institutional investor consultants (hereafter "consultants"), with all data collection details provided in Internet Appendix A. In this subsection, we provide only a brief overview of the dataset.

Our ultimate goal is to better understand the beliefs of institutional investors. As such, the inclusion of asset managers is important. We also include consultants as we argue that their views provide an indirect way to learn about the beliefs of institutional investors. This argument is valid if (i) the beliefs of institutional investors and consultants are simultaneously based on common signals and/or (ii) the beliefs of consultants have a causal impact on the beliefs of institutional investors. While we do not attempt to explore these two channels in our paper, Internet Appendix B shows that our main results are similar when we use only managers or only consultants in our baseline analysis.[3]

We collect the long-term CMAs of 34 institutions in total: 18 managers and 16 consultants.[4] The bulk of the data (82% of the institution-year observations) comes directly from the CMAs of the institutions we cover through direct data requests and/or online searches for CMAs. However, we supplement the direct CMAs of these institutions with indirect data obtained from pension funds through their internal reports.[5] In Internet Appendix B, we show

[3]Andonov et al. (2023) show that the beliefs and portfolio allocations of pension funds are connected to the CMAs of their consultants. As anecdotal evidence for this connection, consider the following quote from the Wall Street Journal article (from May 5, 2023) titled *Pension Funds Consider Unloading Stocks, Adding Credit*: "Board members of the $307 billion California State Teachers' Retirement System voted Thursday to reduce the fund's stockholdings to 38% from 42%, a shift staff and consultants said would lower the fund's risk level without bringing down returns."

[4]Many institutions have large consulting businesses while also managing assets directly or on behalf of their clients. We classify these institutions as consultants or managers based on our judgment of their primary business model. Note that this classification does not affect our main results (since we report results for consultants and managers combined). The classification is only relevant to provide the reader a broad understanding of the institutions in our sample and for robustness checks in which we replicate our findings using only consultants or only managers.

[5]It is common practice for institutional investors like pension funds to list the long-term CMAs of their

that our results are similar whether we rely only on the direct CMAs of these institutions or only on the indirect CMAs we obtain from pension funds, alleviating potential concerns with either data collection approach (e.g., the worry that institutions send us historical projections that rely on their current models/methods).

To comply with the data sharing agreements of these institutions, our analysis does not link specific results to individual managers or consultants. However, Table 1 shows the list of managers and consultants that enter our sample at any time.[6] Our sample covers many of the major asset managers and institutional investor consultants. For instance, our managers have total Assets Under Management (AUM) above $23.6 Trillion at the end of 2021, which is equivalent to more than 26.6% of the total AUM of all the top 50 asset managers in the world (by AUM). Similarly, in a typical year from 2001 to 2021, our consultants include the primary consultant of more than 50% of the US public pension funds, covering more than 65% of the total AUM of US public pension funds.[7]

The first five rows in Panel A of Table 2 provide the number of institutions in our sample by year as well as the split between managers and consultants.[8] To conserve space, we only show years 1987, 1996, 1997, and 1998 to 2022 in steps of two years. From 1987 to 1996, our dataset covers a single institution. However, in 1997 two new institutions enter the dataset,

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consultants in their capital allocation report and/or in their CMA report.

[6]The name of Asset Manager #18 (as well as its respective AUM information) will be provided in Table 1 after approval.

[7]The underlying data of primary consultants for US public pension funds come from Center for Retirement Research at Boston College. Four of our consultants are included in the "All Others" category in this dataset, which means we cannot identify how many pension funds have them as primary consultants in a typical year. As such, the total coverage of US pension funds and their AUM provided in Table 1 is a lower bound on the true coverage of the consultants included in our dataset.

[8]Years are defined based on the approximate timing of the institution's information set. For instance, if a CMA contains $\mathbb{E}_{2000}[R_{2001\rightarrow 2010}]$, then our year variable is 2000.

which then grows over time.[9,10]

Each institution-year CMA covers a range of asset classes. Our final sample contains a risk-free asset class proxy (*US Cash*) as well as 19 risky asset classes.[11] To decide on these 19 risky asset classes, we consider three aspects. First, whether the asset class is a major asset class for institutional investors. Second, whether the asset class is covered by a reasonable number of institutions in our sample. And third, whether the asset class is reasonably covered by the institutions over the time period it is in the sample. We include all asset classes that perform well along these dimensions. Because the asset class names and corresponding indexes can differ both across institutions and over time, we use our judgment when mapping asset classes within and across institutions to the asset classes included in our final sample. Internet Appendix A provides further details.

The last two rows in Panel A of Table 2 show the number of unique asset classes covered in our sample as well as the average number of asset classes per institution. Our sample covers 4 asset classes in 1987 and this number grows steadily, reaching 20 asset classes by 2011 (*US Cash* plus our 19 risky asset classes) and remaining at this level until the end of the sample. There is some variation in the coverage of asset classes across institutions and

[9]Note that the maximum number of institutions in any given year is 24 even though we have 34 unique institutions in our dataset. The reason is that the data for some institutions come from CMAs we obtain online or from pension fund reports, nether of which ensures continuous coverage of a given institution over time. For all data sent to us directly by the underlying institutions, our coverage is continuous (i.e., annual). In fact, the second row of Table 2 Panel A is monotonically increasing over time because it is mostly based on data sent to us directly by the underlying institutions (with direct CMAs obtained online providing a complementary data source). See Internet Appendix A for further details on the data collection process.

[10]In all subsequent tables that describe results by year, we display only years starting in 1998. The reason is that for $t \geq 1998$ we always have at least 4 institutions in any given year, and thus one cannot recover results from a single institution from the annual results reported. Whenever reporting time series averages of these annual values, we include all years starting in 1987 (unless otherwise noted) and we weight each year by the number of observations in that year (typically, the number of institutions in that year). Note that aggregate time-series plots (such as the one in Figure 2) are based on beliefs that are aggregated in a way that also prevents the reader from recover results from any single institution (see Subsection 4.1 for the aggregation method we use).

[11]We use the terminology "risk-free" just to be consistent with the literature. Our proxy for the risk-free asset (*US Cash*) is not truly free of risk for the horizons underlying the CMAs we study. This aspect does not undermine our empirical analysis since *US Cash* merely serves as a baseline asset to allow us to measure expected excess returns.

over time, but since 2005 the average institution covers at least 11 asset classes (*US Cash* plus 10 risky asset classes).

Panel B of Table 2 shows the name of each asset class we cover and the number of institutions that cover it each year. *US Cash* is always covered by all institutions so that we have full availability of our risk-free asset class. We also have four broad risky asset classes: Debt, Equity, Real Estate, and Alternatives. For Debt, we have eight asset classes covering different parts of the debt market, with *US Bonds*, *Global Bonds (Ex US)*, and *Private Debt* being three broad ones and *US TIPS*, *US Govt Bonds*, *US Muni Bonds, US Inv Grade Corp Bonds*, and *US High Yield Corp Bonds* being more specialized ones. For Equities, we have five asset classes, with *US Equities (Large Cap)* and *US Equities (Small Cap)* covering the US public market, *Global Equities (Dev, Ex US)* and *Global Equities (Emerging)* covering the international public market, and *Private Equity* covering the private market. For Real Estate, we have two asset classes, with *REITs* covering the public market and *Private Real Estate* covering the private market. Finally, we have *Hedge Funds*, *Commodities*, *Venture Capital*, and *Infrastructure* as the four asset classes under our broad Alternatives asset class.

Some of our risky asset classes are covered by all institutions present in any given year (e.g., *US Equities (Large Cap)*). However, other asset classes (e.g., *Hedge Funds*) have little coverage in the early years and good coverage starting in the mid 2000s.

Table 3 shows the average values for the belief quantities we observe at the end of 2022 pooled across institutions. We observe the same information for each institution-year observation, but for a subset of these 20 asset classes. As Panel A shows, we observe expected returns, expected volatilities, and expected correlations (which allow us to obtain the subjective covariance matrix of returns).

Our analysis is based on excess returns, so we explore returns on our 19 risky asset classes (indexed by n) in excess of our risk-free asset class (indexed by f). Panel B shows the implied belief quantities for these excess returns ($r_n = R_n - R_f$). Specifically, letting $\mathbb{E}_{j,t}[R]$ and $\Sigma^R_{j,t}$ represent the subjective expected return vector and covariance matrix for institution j at time t (with *US Cash* as the first asset class), Panel B reports the 2022 average values for the expected excess returns ($\mu_{j,n,t} \equiv \mathbb{E}_{j,t}[r_n] = \mathbb{E}_{j,t}[R_n] - \mathbb{E}_{j,t}[R_f]$) and covariance matrices of excess returns ($\Sigma_{j,t} = \Omega\Sigma^R_{j,t}\Omega'$, where $\Omega = [-1, \mathrm{I}]$, with 1 representing a column vector of ones and I an identity matrix). Our analysis is based on these $\mu_{j,n,t}$ and $\Sigma_{j,t}$ values.[12] For brevity, in the rest of the text we refer to $\mu_{j,n,t}$ as the expected return on asset class n (instead of expected excess return on asset class n).

Before detailing our main empirical results, it is important to point out that our analysis in the main text is based on institutions' expected arithmetic (excess) returns and without any homogeneity restriction on the horizon of beliefs (i.e., different institutions report beliefs for different horizons in the baseline analysis). In Internet Appendix B, we show that our conclusions are very similar if we (i) use expected geometric returns to measure $\mathbb{E}[R]$, or (ii) focus only on institution-year observations with a horizon of 10 years, which is the most common horizon in our dataset (capturing around 41% of the institution-year observations).

2.3 A Validation Check of our Data

As a preliminary analysis of our data, Figure 2 plots the subjective equity premium aggregated across all institutions each year (based on *US Equities (Large Cap)*) together with the S&P 500 earnings yield (i.e., the log of the inverted CAPE ratio from Shiller), which Dahlquist and Ibert (2023) use to identify the equity premium cyclicality in their data.[13] As it is clear from the figure, the subjective equity premium is highly countercyclical. More specifically, its correlation with the S&P 500 earnings yield is around 0.60 despite the fact that this correlation is biased towards zero because we do not know the exact dates of the CMA information sets for many institution-year observations (so, we align the two times series at the end of December of each year). This result serves as a validation check since it

[12] Note that the reported average $\mathbb{E}_t[r_n]$ is not exactly equal to the average reported $\mathbb{E}_t[R_n]$ minus the average reported $\mathbb{E}_t[R_f]$. The reason is that the excess returns are calculated at the institution-year level, and the data coverage for *US Cash* (our R_f) can differ from the data coverage of the given risky asset class, with the mismatch varying across institutions.

[13] Our aggregate subjective equity premium accounts for differences in sample composition over the years. The aggregation methodology details are provided in the header of Figure 2, with more details and discussion in Subsection 4.1.

confirms one of the main results in Dahlquist and Ibert (2023), who study expected returns from CMAs. Figure 2 also extends the findings in Dahlquist and Ibert (2023) to a longer time series, which helps to properly identity the cyclicality of the subjective equity premium (note that the sample in Dahlquist and Ibert (2023) is sparse before 2010).

3 The Subjective Risk-Return Tradeoff

In this section, we explore the link between subjective risk and return expectations. Subsection 3.1 shows that (i) the subjective market risk premia are consistent with the subjective expected returns on the respective market portfolios and (ii) the overall variation in subjective expected returns is mostly driven by variation in subjective risk premia (compensation for beta), with subjective alphas having only a secondary role. Subsection 3.2 demonstrates that most of the variation in subjective expected returns and betas originates from variation across asset classes with variation across institutions (disagreement) playing only a secondary role (albeit, a non-trivial role).

3.1 Subjective Expected Returns: Risk Premia vs Alphas

To start, we need a model to determine the risk premium of each asset class under the subjective measure of each institution. For this purpose, we focus on the Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965):

$$\mu_{j,n,t} = \lambda_{j,t}^{(0)} + \beta_{j,n,t}^{(m)} \cdot \lambda_{j,t}^{(m)} + a_{j,n,t}^{(m)} \tag{5}$$

with model implications $\lambda_{j,t}^{(0)} = 0$, $\lambda_{j,t}^{(m)} = \mu_{j,t}^{(m)}$, and $a_{j,n,t}^{(m)} = 0$. Note that the superscript (m) refers to the market portfolio and takes two different values below $(m = e, p)$ to reflect two alternative proxies for the market portfolio.

Given a proxy for the market portfolio weights $(w_{m,t})$, we can calculate $\beta_{j,n,t}^{(m)} = \mathbb{C}ov_{j,t}[r_n, r_m]/\mathbb{V}ar_{j,t}[r_m] = 1_n' \Sigma_{j,t} w_{m,t}/w_{m,t}' \Sigma_{j,t} w_{m,t}$. However, as pointed out by Roll (1977), it is infeasible to obtain a perfect proxy for the market portfolio weights. With this issue in mind, we consider two alternative empirical versions of the CAPM (i.e., two alternative market proxies). The first is the Equity CAPM ($m = e$), with the market portfolio based on *US Equities (Large Cap)*. The second is the Pension CAPM ($m = p$), with the market portfolio based on the aggregate allocations of US Public Pension Funds.[14] The main advantages of the Equity CAPM are that (i) its market proxy is common in much of the literature that tests the CAPM using realized returns and (ii) we observe beliefs for *US Equities (Large Cap)* for all our institution-year observations. The downside of the Equity CAPM is that it relies on a market proxy that may not reflect the wealth portfolio of institutional investors. Our Pension CAPM addresses this issue as it considers a much broader market portfolio, with weights on several asset classes.[15] The downside of this approach is that some institution-year observations do not have beliefs for some of the asset classes included in the aggregate portfolio of pension funds. In these cases, we adjust the market portfolio weights so that they continue to sum to one (effectively allocating the weights for the missing asset class to the other asset classes in the portfolio).

We obtain $\lambda_{j,t}^{(m)}$ by projecting $\mu_{j,n,t}$ onto $\beta_{j,n,t}^{(m)}$ in the cross-section of asset classes for each institution-year observation. Figure 3 (Panels (a) to (d)) and Table 4 are designed to explore the $\lambda_{j,t}^{(m)} = \mu_{j,t}^{(m)}$ implication (i.e., to test whether institutional investors require a risk premium on β that is quantitatively consistent with their subjective expected return on the underlying market factor). In particular, Figure 3 reports the distributions (across institution-year observations) of $\lambda_{j,t}^{(m)}$ and $\mu_{j,t}^{(m)}$ displayed in different ways while Table 4 reports the distribution of $\lambda_{j,t}^{(m)} - \mu_{j,t}^{(m)}$ with their respective t-statistics (based on $\lambda_{j,t}^{(m)}$ sampling error)

[14]An alternative interpretation for our Pension CAPM is that it reflects an asset pricing model based on the wealth portfolio of asset managers. If asset managers rely on a mean-variance framework, then we have an SDF that is linear in the wealth portfolio of asset managers even if there is no SDF that is linear in the market portfolio (which combines the wealth portfolio of asset managers with the wealth portfolio of other investors).

[15]Internet Appendix Figure IA.1 shows the market portfolios weights for our Pension CAPM from 2001 to 2021 (we use 2001 weights for earlier years and 2021 weights from later years). The data comes from the Center for Retirement Research at Boston College. The average weights are 2% for Cash (matched to *US Cash*), 34% for US Equities (matched to *US Equities (Large Cap)*), 20% for Global Equities (matched to *Global Equities (Dev, Ex US)* and, if missing, *US Equities (Large Cap)*), 7% for Private Equity (matched to *Private Equity*), 24% for US Bonds (matched to *US Bonds* and, if missing, *US Govt Bonds*), 3% for Global Bonds (matched to *Global Bonds (Ex US)* and, if missing, *US Bonds*), 7% for Real Estate (matched to *REITs* and, if missing, *Private Real Estate*), and 4% for Hedge Funds (matched to *Hedge Funds*).

for each year.

The Equity CAPM results indicate that, for the typical institution-year observation, the pricing of market betas is consistent with the subjective expected return on the market portfolio (i.e., the difference between $\lambda_{j,t}^{(e)}$ and $\mu_{j,t}^{(e)}$ tends to be small and statistically insignificant). Interestingly, the largest median deviation in the post 1996 period (when we have multiple institutions in the sample) is -0.7% and happens in the middle of the great recession (2008), but even then the difference for the typical institution is not statistically significant once we account for the sampling error in $\lambda_{j,t}^{(e)}$. The results are similar for the Pension CAPM in that median deviations each year tend to be relatively small. One difference, however, is that the median deviations tend to be negative (on average, -0.7% with $t_{stat} = -1.16$). In fact, in 2008 the median deviation is statistically significant (albeit not that large economically, at -1.3%).

Figures 3(e) and 3(f) explore the $\lambda_{j,t}^{(0)} = 0$ implication from the CAPM (that the subjective expected return on the risk-free asset is consistent with the pricing of β). Specifically, these figures plot the distributions (across institution-year observations) of $\lambda_{j,t}^{(0)}$ for the Equity CAPM and Pension CAPM. The overall results indicate that the restriction $\lambda_{j,t}^{(0)} = 0$ does not hold in the subjective belief space of the institutions we study. In particular, we find $\lambda_{j,t}^{(0)} > 0$ for almost all institution-year observations. This result is inconsistent with the baseline CAPM holding in the belief space of our institutions. However, if the institutions perceive constraints in their ability to borrow at the risk-free rate (proxied with *US Cash* in our analysis), then the results remain consistent with the Black (1972) CAPM, which considers investors who cannot borrow at the risk-free rate, implying $\lambda_{j,t}^{(0)} > 0$ (the $\lambda_{j,t}^{(0)} > 0$ result is also consistent with the concept of convenience yield discussed in the banking and macro literatures). As such, we interpret this evidence as suggesting that institutions have beliefs that are largely internally consistent with the pricing of risk implied by the CAPM.

While the above results explore whether subjective expected returns are consistent with the pricing of systematic risk, these results do not tell us the sizes of the CAPM pricing errors in Equation 5. To explore this aspect, we note that Equation 5 can alternatively be written as (imposing $\lambda_{j,t}^{(0)} = 0$ and $\lambda_{j,t}^{(m)} = \mu_{j,t}^{(m)}$)

$$\mu_{j,n,t} \;=\; \alpha_{j,n,t}^{(m)} \;+\; \beta_{j,n,t}^{(m)} \cdot \mu_{j,t}^{(m)} \;=\; \alpha_{j,n,t}^{(m)} \;+\; rp_{j,n,t}^{(m)} \tag{6}$$

where $rp_{j,n,t}^{(m)} = \beta_{j,n,t}^{(m)} \cdot \mu_{j,t}^{(m)}$ with the CAPM implying $\alpha_{j,n,t}^{(m)} = 0$.

Figures 4(a) and 4(b) show the alphas for each asset class from Equation 6 averaged over time and across institutions (with each $\alpha_{j,n,t}$ calculated directly from Equation 6 since we observe all other quantities in this equation). The striking result is that even though there is large variation in subjective expected returns across asset classes, there is little variation in alphas and such variation has little relation to the variation in expected returns. These results suggest that differences in subjective expected returns across asset classes are largely driven by differences in subjective risk premia, not subjective alphas. As such, the CAPM reasonably describes subjective expected return variation across asset classes.

Figures 4(a) and 4(b) also show that average alphas do not center around zero. That is, there is a tendency for positive alphas irrespective of the underlying asset class. This result is a direct consequence of the $\lambda_{j,t}^{(0)} > 0$ finding. To demonstrate this, Figures 4(c) and 4(d) show the average unrestricted pricing errors (a) from Equation 5 (which allow for $\lambda_{j,t}^{(0)} > 0$). In this case, pricing errors do center around zero and it remains the case that variation in subjective expected returns across asset classes is largely explained by variation in subjective risk premia.

While Figure 4 displays asset class pricing errors after averaging over time and across institutions, Figure 5 explores all alpha values (i.e., with no averaging). Specifically, Figures 5(a) and 5(b) plot $\mu_{j,n,t}$ against $rp_{j,n,t}^{(m)} = \beta_{j,n,t}^{(m)} \cdot \mu_{j,t}^{(m)}$ whereas Figures 5(c) and 5(d) plot $\mu_{j,n,t}$ against $\alpha_{j,n,t}^{(m)}$. The results are striking as they suggest that $\mu_{j,n,t}$ varies much more with $rp_{j,n,t}^{(m)}$ than with $\alpha_{j,n,t}^{(m)}$ in both versions of the CAPM.

To explore a more formal decomposition of expected return variation, we take variance on both sides of Equation 6 to get:

$$\begin{aligned} \mathbb{V}ar\left[\mu_{j,n,t}\right] &= \mathbb{C}ov\left[\mu_{j,n,t}\ ,\ \alpha_{j,n,t}\right] \quad + \quad \mathbb{C}ov\left[\mu_{j,n,t}\ ,\ rp_{j,n,t}\right] \\ &\Downarrow \\ 1 &= \underbrace{\frac{\mathbb{C}ov\left[\mu_{j,n,t}\ ,\ \alpha_{j,n,t}\right]}{\mathbb{V}ar\left[\mu_{j,n,t}\right]}}_{\text{\% of } \mu \text{ Variation from Mispricing}} + \underbrace{\frac{\mathbb{C}ov\left[\mu_{j,n,t}\ ,\ rp_{j,n,t}\right]}{\mathbb{V}ar\left[\mu_{j,n,t}\right]}}_{\text{\% of } \mu \text{ Variation from Risk Premia}} \end{aligned} \tag{7}$$

Table 5 provides results associated with the decomposition in Equation 7, where the first term is obtained from a projection of $\alpha_{j,n,t}$ onto $\mu_{j,n,t}$ and the second term is obtained from a projection of $rp^{(m)}_{j,n,t} = \beta^{(m)}_{j,n,t} \cdot \mu^{(m)}_{j,t}$ onto $\mu_{j,n,t}$. Different specifications add different fixed effects to the projections. In column [1], we find that around 75% (90%) of the variability in μ comes from variation in risk premia under the Equity CAPM (Pension CAPM). Columns [2] and [3] show that adding year or institution fixed effects has basically no effect on these results. However, Column [4] shows that adding asset class fixed effects largely changes the result, with variation in alphas becoming non-trivial as it explains almost 50% (40%) of the variability in μ in the Equity CAPM (Pension CAPM). This result indicates that it is informative to explore different sources of variation in the data. Columns [5] and [6] focus on variation across asset classes only and effectively reproduce the baseline result that around 75% (90%) of the variability in μ comes from variation in risk compensation under the CAPM (Pension CAPM). As such, when we observe two asset classes having different subjective expected returns, we can largely conclude that it comes from the fact that managers perceive these asset classes as requiring different risk premia. Columns [7] and [8] focus on variation across institutions and find that in this case alphas explain around 50% (40%) of the variability in μ under the Equity CAPM (Pension CAPM). As such, when we observe two institutions disagreeing about the expected return for the same asset class, a substantial portion of the difference comes from the fact that these institutions perceive different risk premia for this asset class, but a non-trivial component of the difference arises from institutions perceiving different alphas for this asset class. Finally, Columns [9] and

[10] focus on variation over time and also find that time-variation in alphas has a non-trivial quantitative effect.

3.2 Belief Heterogeneity Across Asset Classes and Institutions

The prior subsection shows that variation in subjective expected returns across asset classes is largely driven by variation in subjective risk premia whereas subjective alphas have a non-trivial role in explaining variation in subjective expected returns across institutions for a given asset class. As such, to fully understand the importance of alphas, we need to understand the importance of belief heterogeneity across asset classes and across institutions, which is the focus of this subsection.

To start, we decompose within-year variability in expected returns into variability originating from institutions and variability originating from asset classes. Specifically, we estimate the fixed-effects model

$$\mu_{j,n,t} = \overline{\mu}_{j,t} + \overline{\mu}_{n,t} + \eta_{j,n,t} \tag{8}$$

where $\overline{\mu}_{j,t}$ and $\overline{\mu}_{n,t}$ are year×institution and year×asset class fixed effects and $\eta_{j,n,t}$ are residuals.

Figures 6(a) and 6(b) plot $\mu_{j,n,t}$ against $\overline{\mu}_{j,t}$ and $\overline{\mu}_{n,t}$, respectively. Figure 6(a) shows that there is only a weak positive relation between $\overline{\mu}_{j,t}$ and $\mu_{j,n,t}$. Moreover, for any given level of $\overline{\mu}_{j,t}$ we have a high degree of $\mu_{j,n,t}$ heterogeneity, indicating that only a small fraction of the variability in $\mu_{j,n,t}$ is due to heterogeneity across institutions (captured by $\overline{\mu}_{j,t}$). In contrast, Figure 6(b) shows a strong positive relation between $\overline{\mu}_{n,t}$ and $\mu_{j,n,t}$, with a more moderate degree of $\mu_{j,n,t}$ heterogeneity at each level of $\overline{\mu}_{n,t}$. Consequently, a large fraction of the variability in $\mu_{j,n,t}$ is due to heterogeneity across asset classes (captured by $\overline{\mu}_{n,t}$).

To precisely quantify the patterns observed in Figures 6(a) and 6(b), we take variance on both sides of Equation 8 to obtain the following decomposition:

$$1 = \underbrace{\frac{\mathbb{C}ov\left(\mu_{j,n,t}\ ,\ \overline{\mu}_{j,t}\right)}{\mathbb{V}ar(\mu_{j,n,t})}}_{\%\text{ of }\mu_{j,n,t}\text{ Variation from }\overline{\mu}_{j,t}} + \underbrace{\frac{\mathbb{C}ov\left(\mu_{j,n,t}\ ,\ \overline{\mu}_{n,t}\right)}{\mathbb{V}ar(\mu_{j,n,t})}}_{\%\text{ of }\mu_{j,n,t}\text{ Variation from }\overline{\mu}_{n,t}} + \underbrace{\frac{\mathbb{C}ov\left(\mu_{j,n,t}\ ,\ \eta_{j,n,t}\right)}{\mathbb{V}ar(\mu_{j,n,t})}}_{\%\text{ of }\mu_{j,n,t}\text{ Variation from }\eta_{j,n,t}} \tag{9}$$

Figure 7(a) provides the decomposition in Equation 9 each year. The results indicate that the vast majority of within-year expected return variability is driven by variability across asset classes ($\overline{\mu}_{n,t}$), with only a small portion driven by variability across institutions ($\overline{\mu}_{j,t}$). This result is striking and explains why expected return variation is largely driven by risk premia and not alphas. Specifically, while subjective alphas are important in explaining subjective expected return variation across institutions (for a given asset class), the variation in subjective expected returns across institutions (driven by disagreement) is small relative to the variation in subjective expected returns across asset classes (driven by the positive risk-return tradeoff).

One limitation of the baseline econometric model in Equation 8 (and its respective variance decomposition in Equation 9) is that it does not allow institutions to have expected return heterogeneity that differs across asset classes. For example, if institution j is relatively optimistic about *US Bonds* at time t, then it is also relatively optimistic about all other asset classes at time t (i.e., $\overline{\mu}_{j,t}$ is common across asset classes). To address this issue, we also consider the alternative fixed-effects model

$$\mu_{j,n,t} = \overline{\mu}_{n,j} + \overline{\mu}_{n,t} + \eta_{j,n,t} \tag{10}$$

which leads to the variance decomposition

$$1 = \underbrace{\frac{\mathbb{C}ov\left(\mu_{j,n,t}\ ,\ \overline{\mu}_{n,j}\right)}{\mathbb{V}ar(\mu_{j,n,t})}}_{\%\text{ of }\mu_{j,n,t}\text{ Variation from }\overline{\mu}_{n,j}} + \underbrace{\frac{\mathbb{C}ov\left(\mu_{j,n,t}\ ,\ \overline{\mu}_{n,t}\right)}{\mathbb{V}ar(\mu_{j,n,t})}}_{\%\text{ of }\mu_{j,n,t}\text{ Variation from }\overline{\mu}_{n,t}} + \underbrace{\frac{\mathbb{C}ov\left(\mu_{j,n,t}\ ,\ \eta_{j,n,t}\right)}{\mathbb{V}ar(\mu_{j,n,t})}}_{\%\text{ of }\mu_{j,n,t}\text{ Variation from }\eta_{j,n,t}} \tag{11}$$

Figures 6(c) and 6(d) show that the results from Equation 10 are similar to the results from Equation 8. That is, $\overline{\mu}_{n,j}$ has a relatively weak relation with $\mu_{j,n,t}$ whereas $\overline{\mu}_{n,t}$ has a strong relation with $\mu_{j,n,t}$. Moreover, Figure 7(b) implements the decomposition in Equation 11. While the institution effect (through $\overline{\mu}_{n,j}$) explains more of the expected return variability than it does in Equation 9 (through $\overline{\mu}_{j,t}$), it remains true that the majority of the expected return variability is driven by the asset class effect ($\overline{\mu}_{n,t}$).

We can generalize the analysis above to any other belief quantity (generically referred to as θ). Specifically, we can decompose the within-year variability in θ into variability originating from institutions and variability originating from asset classes through the generic fixed effects model

$$\theta_{j,n,t} = \overline{\theta}_{j,t} + \overline{\theta}_{n,t} + \eta_{j,n,t} \tag{12}$$

which leads to the variance decomposition

$$1 = \underbrace{\frac{\mathbb{C}ov\left(\theta_{j,n,t}\ ,\ \overline{\theta}_{j,t}\right)}{\mathbb{V}ar(\theta_{j,n,t})}}_{\%\text{ of } \theta_{j,n,t} \text{ Variation from } \overline{\theta}_{j,t}} + \underbrace{\frac{\mathbb{C}ov\left(\theta_{j,n,t}\ ,\ \overline{\theta}_{n,t}\right)}{\mathbb{V}ar(\theta_{j,n,t})}}_{\%\text{ of } \theta_{j,n,t} \text{ Variation from } \overline{\theta}_{n,t}} + \underbrace{\frac{\mathbb{C}ov\left(\theta_{j,n,t}\ ,\ \eta_{j,n,t}\right)}{\mathbb{V}ar(\theta_{j,n,t})}}_{\%\text{ of } \theta_{j,n,t} \text{ Variation from } \eta_{j,n,t}} \tag{13}$$

Figure 8 provides results for the decomposition in Equation 13 applied to βs and αs (for the Equity CAPM and Pension CAPM). In a nutshell, the results indicate that almost all the variability in βs is driven by variability across asset classes. In contrast, disagreement plays a large role in explaining alpha variability, with disagreement ($\overline{\alpha}_{j,t} + \eta_{j,n,t}$) being roughly as important as variation across asset classes ($\overline{\alpha}_{n,t}$) in explaining overall alpha variation.

4 Subjective Beliefs vs Realized Returns

In this section, we explore the link between subjective beliefs and realized returns. Subsection 4.1 describes how we aggregate the beliefs data across institutions and provides information about the realized return data we rely on. Subsection 4.2 contrasts the risk-return tradeoff implied from the subjective beliefs of our institutions with the risk-return tradeoff observed in the realized return data. Subsection 4.3 shows that, on average, beliefs are largely consistent with their realized return counterparts, but points out some exceptions. Subsection 4.4 provides an analysis of the predictability of realized return moments using subjective beliefs.

4.1 Belief Aggregation and Return Data

To explore the link between beliefs and realized returns, we need beliefs aggregated across institutions. We aggregate beliefs by estimating the fixed effects regression

$$\theta_{j,n,t} \;= \overline{\theta}_{n,j} \;+\; \overline{\theta}_{n,t} \;+\; \eta_{j,e,t} \tag{14}$$

and then obtaining the aggregate belief quantity

$$\theta_{n,t} \;=\; \left[\frac{1}{T}\cdot\sum_{t=1}^{T}\left(\frac{1}{J_t}\sum_{j=1}^{J_t}\theta_{j,n,t}\right)\right] \;+\; \left(\overline{\theta}_{n,t} \;-\; \frac{1}{T}\cdot\sum_{t=1}^{T}\overline{\theta}_{n,t}\right) \tag{15}$$

where $\theta_{j,n,t}$ represents a generic belief quantity (μ, σ, $\mu^{(m)}$, $\beta^{(m)}$).[16] In turn, we obtain aggregate alphas using $\alpha^{(m)}_{n,t} = \mu_{n,t} - \beta^{(m)}_{n,t} \cdot \mu^{(m)}_{t}$.[17,18]

Intuitively, $\overline{\theta}_{n,t}$ captures time variation in beliefs controlling for time variation in the composition of institutions (through the $\overline{\theta}_{n,j}$ fixed effects). However, the average $\overline{\theta}_{n,t}$ cannot be interpreted as the unconditional mean for the belief quantity. As such, our aggregation takes the time demeaned $\overline{\theta}_{n,t}$ and adds to it the time-series average of the cross-section averages of $\theta_{j,n,t}$, which is an estimate for $\mathbb{E}[\theta_n]$.

Note that the time-series average of $\theta_{n,t}$ matches the time-series average of the alternative aggregated series $\frac{1}{J_t}\sum_{j=1}^{J_t}\theta_{j,n,t}$. However, our aggregation method underlying $\theta_{n,t}$ is designed to produce time variation that accounts for sample composition (whereas $\frac{1}{J_t}\sum_{j=1}^{J_t}\theta_{j,n,t}$ is not). The reason is that aggregating without accounting for sample composition would induce time variation in beliefs that is purely due to variation in the set of institutions providing the given belief quantity (i.e., it would exist even if beliefs were constant over time within each institution).

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[16]In the case of $\beta^{(p)}_{j,n,t}$, the asset classes in the market portfolio can vary across j (because some institutions do not cover all asset classes present in the pension market portfolio). As such, instead of directly aggregating $\beta^{(p)}_{j,n,t}$ for each year, we aggregate the covariance for each pair of asset classes and combine aggregate variances and covariances with the pension market portfolio weights to obtain $\beta^{(p)}_{n,t}$. However, our results are similar if we directly aggregate $\beta^{(p)}_{j,n,t}$.

[17]Directly aggregating $\alpha^{(m)}_{j,n,t}$ yields similar $\alpha^{(m)}_{n,t}$ values, but we opt for using $\alpha^{(m)}_{n,t} = \mu_{n,t} - \beta^{(m)}_{n,t}\cdot\mu^{(m)}_{t}$ so that we still have the equation $\mu_{n,t} = \alpha^{(m)}_{n,t} + \beta^{(m)}_{n,t}\cdot\mu^{(m)}_{t}$, with the CAPM implying $\alpha^{(m)}_{n,t} = 0$.

[18]Note that Figure 2 (discussed in Subsection 2.3) plots $\mu_{e,t}$ based on this aggregation method.

Also note that our aggregation method partially accounts for the fact that institutions have heterogeneous forecasting horizons and CMA report dates (i.e., CMAs are produced around December of each year but not exactly at the end of December of each year).[19] Forecasting horizon and reporting dates play a role when we attempt to forecast returns. We explain how we deal with that in Subsection 4.4.

Given aggregate beliefs, we also need realized returns for each of the asset classes underlying our beliefs data. Some institutions provide tradable indices as references for the different asset classes in their CMAs while other institutions do not. Moreover, for the ones that do provide tradable indices, two institutions may use different indices for the same asset class. So, the matching between beliefs and return data is bounded to be imperfect. We obtain return data for 17 out of the 20 asset classes we study (leaving out *Private Equity*, *Private Debt*, and *Venture Capital*).[20] Table 6 provides the details for the return data we use, including the names of the indices, the data sources, and the sample period over which we observe returns for each index. We obtain monthly returns for each index (quarterly returns for *Private Real Estate*) and compound these returns to create time series of annual returns on a monthly frequency (quarterly frequency for *Private Real Estate*).

The following subsections use these annual returns or data moments obtained from these annual returns. In particular, the realized return versions of μ_n, σ_n^2, $\beta_n^{(m)}$, and $\mu^{(m)}$ are simply the sample estimates of these quantities using the time-series of annual returns for each asset class. Moreover, $rp_n^{(m)} = \beta_n^{(m)} \cdot \mu^{(m)}$ and $\alpha_n^{(m)} = \mu_n - rp_n^{(m)}$ are analogous to their belief counterparts. Hereafter, we use hats to denote data moments. For instance, $\widehat{\mu}_n$ is the

[19] Specifically, the year-t CMA of institution j is meant to represent institution j forecast for the period from $t+1$ to $t+h_j$ (where h_j is the institution forecasting horizon). Moreover, it is not produced exactly at the end of December of year t, but rather around that date, with some institutions having reports dated as early as September of year t and other institutions having reports dated as late as March of year $t+1$ (within the sample of institution-year observations for which we observe the CMA report date). To the extent that h_j and the reporting date gap (relative to December) is fixed over time for each institution, our $\overline{\theta}_{n,j}$ fixed effects account for that. Time variation in horizons and reporting gaps are not controlled for in our aggregation method, but should be of second-order importance (and would add noise to our belief data, deteriorating the match between beliefs and return data, leading to the opposite of our main findings).

[20] We leave out *Private Equity*, *Private Debt*, and *Venture Capital* because of the difficulty in obtaining quality data on the returns on these asset classes over our sample period.

average return for asset class n.

4.2 Risk-Return Tradeoff: Subjective Beliefs vs Realized Returns

This subsection contrasts the risk-return tradeoff implied by the subjective beliefs of our institutions with the risk-return tradeoff observed in the return data.

Figure 9 plots expected returns (μ_n) against risk premia ($rp_n^{(m)}$) for our asset classes using both beliefs and realized returns. Figures 9(a) and 9(b) are analogous to Figures 5(a) and 5(b), except that they rely on μ_n and $rp_n^{(m)}$ values aggregated across institutions and over time (and thus only vary across asset classes). Not surprisingly, these figures uncover the strong subjective risk-return tradeoff we emphasize in this paper. Figures 9(c) and 9(d) repeat this analysis but using $\widehat{\mu}_n$ and $\widehat{rp}_n^{(m)}$. These figures demonstrate that the risk-return tradeoff observed in the return data is almost as strong as the subjective risk-return tradeoff of our institutions. It is important to point out that we expect the risk-return tradeoff from realized returns to be weaker (i.e., the R^2 values from the figures to be smaller) because average returns provide noisy estimates for unconditional expected returns (Fama and French (2002)).

Another way to look at the risk-return tradeoff is to check whether betas predict alphas across asset classes. Within each asset class, this is known as the "low-beta anomaly" (see Frazzini and Pedersen (2014)). Figures 10(a) and 10(b) show that there is no low-beta anomaly across asset classes in the subjective beliefs of our institutions. Figures 10(c) and 10(d) show that, in the return data, there is a small low-beta anomaly across asset classes. This pattern indicates that beliefs are not fully consistent with realized returns. However, the discrepancy is so small that it has little impact on the overall message that the subjective risk-return tradeoff is consistent with the risk-return tradeoff observed in the realized return data.

4.3 Consistency Between Subjective Beliefs and Realized Returns

This subsection shows that, on average, beliefs are largely consistent with their realized return counterparts. One exception, however, is that α_n values are not consistent with $\widehat{\alpha}_n$ values obtained from subsequent returns even though they are consistent with the full sample $\widehat{\alpha}_n$ values (i.e., alphas that use return data starting before the belief formation).

Figure 11 plots realized return moments against averages of belief quantities. As in the prior subsection, we use the entire return time-series for each asset class (so, returns start at the "First Return Date" column of Table 6). The objective of Figure 11 is to understand whether beliefs are consistent with long-run return data moments, which tend to be less noisy estimates of unconditional moments (than short-run return data moments). Figures 11(a) and 11(b) show that $\widehat{\mu}_n$ and $\widehat{\sigma}_n$ are largely consistent with μ_n and σ_n. For instance, the slope coefficient from regressing $\widehat{\mu}_n$ on μ_n is 0.93 (close to one) and $R^2 = 70\%$ (considering that $\widehat{\mu}_n$ contains substantial estimation noise, this R^2 value is extremely high). The link between $\widehat{\sigma}_n$ and σ_n is even stronger, with a slope coefficient of 1.05 and $R^2 = 90\%$. We observe similar results for betas. However, alphas have a much weaker link. For instance, a regression of $\widehat{\alpha}^{(e)}$ on $\alpha^{(e)}$ results in $R^2 = 34\%$ (albeit the slope coefficient is 0.96, and thus still close to one).

While using the entire return time-series for each asset class minimizes estimation noise in return moments (rendering R^2 values more reliable), it has a key drawback. Specifically, if institutions obtain their beliefs by simply estimating past data moments, the relation between beliefs and data moments will reflect a "consistency with historical data". Asymptotically, this consistency with historical data ensures that beliefs are unconditionally rational (i.e., unconditional beliefs match unconditional return moments). However, in finite samples, this consistency with historical data can induce temporary extrapolation.

To understand whether Figure 11 reflects more than this "consistency with historical data" effect, Figure 12 considers return moments estimated using returns that are subsequent to their respective beliefs. For instance, in Figure 12, we obtain μ_e as the average of $\mu_{e,t}$ from 1987 to 2021 and $\widehat{\mu}_e$ as the average of $r_{e,t}$ from 1988 to 2022 (and similarly for all other quantities over their respective sample periods). As we can see from Figure 12(a), there is still a strong link between average returns and average expected returns, with a regression of $\widehat{\mu}_e$ on μ_e resulting in a slope coefficient of 0.85. The R^2 value declines to 43%, but this result is expected given that the $\widehat{\mu}_n$ values used in Figure 12 have more estimation noise than the $\widehat{\mu}_n$ values used in Figure 11. We can clearly see this aspect from the fact that $\widehat{\mu}_{Commodities} < -3\%$ in Figure 12, which is likely a consequence of estimation noise (i.e., it reflects unexpected returns that did not average to zero over the relatively small sample period). In fact, just dropping $\widehat{\mu}_{Commodities}$ from Figure 12(a) increases the R^2 to 63%. Figures 12(b) to 12(d) further show that we continue to observe a tight link between subjective beliefs and return moments related to risk (volatilities and betas). Interestingly, in this case the R^2 values remain at similar levels. This result is likely a consequence of the fact that variances and covariances (underlying $\widehat{\sigma}_n$ and $\widehat{\beta}_n$) have much less estimation noise than average returns (see, e.g., Merton (1980)). So, we can conclude that the results in Figure 11 reflect more than a "consistency with historical data" effect.

In stark contrast to the paragraph above, Figures 12(e) and 12(f) show that $\widehat{\alpha}_n$ has effectively no connection to α_n. This indicates that the alpha connection in Figure 11 indeed reflects a pure "consistency with historical data" effect. That is, institutions estimate alphas based on past data. Since alphas do not tend to persist in the return data (if the CAPM is correct, alphas are zero over long samples), there is no link between α_n and $\widehat{\alpha}_n$ calculate from subsequent returns. So, in this case, α_n reflects a temporary extrapolation. It is temporary because as historical data accumulates estimated alphas converge to zero (under the CAPM), which would induce beliefs to also converge to zero (if they indeed reflect past alphas), leading to beliefs that are unconditionally rational.

4.4 Subjective Beliefs Predicting Realized Returns and Risk

While the prior subsection focuses on the unconditional link between beliefs and return moments, this section explores the conditional link between beliefs and return moments. We find that subjective expected returns predict subsequent realized returns both across asset classes and over time. Moreover, the quantitative link between subjective expected returns and subsequent realized returns is consistent with rational expectations. We also find that subjective risk predicts realized risk across asset classes but not over time, and that subjective alphas do not predict subsequent realized alphas.

If our institutions, on average, have rational expectations, then we have $\mathbb{E}_{o,t}[r_{n,t+1}] = \mathbb{E}_t[r_{n,t+1}] = \mu_{n,t}^{(1y)}$, which implies

$$r_{n,t+1} = a + b \cdot \mu_{n,t}^{(1y)} + \epsilon_{n,t+1} \tag{16}$$

and

$$1/T_n \cdot \sum_{t=1}^{T_n} r_{n,t+1} = a + b \cdot \left(1/T_n \cdot \sum_{t=1}^{T_n} \mu_{n,t}^{(1y)}\right) + u_n \tag{17}$$

where $a = 0$ and $b = 1$.

We do not observe $\mu_{n,t}^{(1y)}$ (the 1-year subjective expected return), but we do observe $\mu_{n,t}$ (the subjective expected average annual return over the next H years). Note that $1/T_n \cdot \sum_{t=1}^{T_n} \mu_{n,t}^{(1y)}$ and $1/T_n \cdot \sum_{t=1}^{T_n} \mu_{n,t}$ are asymptotically equivalent (as $T_n \to \infty$), and thus Equation 17 continues to hold if we replace $\mu_{n,t}^{(1y)}$ with $\mu_{n,t}$ (in fact, Figure 12(a) performs exactly this regression). Moreover, if $\mu_{n,t}^{(1y)}$ follows an AR(1), then $\mu_{n,t}$ and $\mu_{n,t}^{(1y)}$ are perfectly positively correlated, and thus we can also replace $\mu_{n,t}^{(1y)}$ with $\mu_{n,t}$ and Equation 16 continues to hold (except that we no longer have the $a = 0$ and $b = 1$ implications in that equation).

Table 7 (Panel A, first block) estimates Equation 16 using annual data with $\mu_{n,t}$ as a replacement for $\mu_{n,t}^{(1y)}$. The results indicate that $\mu_{n,t}$ is a strong predictor of $r_{n,t+1}$, with $b = 1.22$ ($t_{stat} = 2.69$) and a moderate $R^2 = 3.8\%$. When we focus only on variation across asset classes (by adding year fixed effects) or only on variation over time (by adding asset class fixed effects), we observe a slight increase in (within) R^2 values (to 4.9% and 4.6% respectively). Interestingly, in the variation across asset classes, we have $b = 1.12$ ($t_{stat} = 2.04$), which is statistically indistinguishable from 1. This happens because a regression with time fixed effects is similar to Equation 17, and thus the $b = 1$ restriction is approximately valid. In contrast, in the variation over time, we have $b = 4.83$ ($t_{stat} = 1.90$), which is far from 1. In the variation over time, the $b = 1$ restriction is far from valid. The reason is that, due to mean reversion, the time-variation in long-run expected returns ($\mu_{n,t}$) is much smaller than the respective time variation in 1-year expected returns ($\mu_{n,t}^{(1y)}$), so that replacing $\mu_{n,t}^{(1y)}$ with $\mu_{n,t}$ in Equation 16 induces $b > 1$.

To check whether the predictability we observe in the time series is consistent with the $b = 1$ implication from Equation 16, we construct a proxy for $\mu_{n,t}^{(1y)}$. Specifically, we note that if $\mu_{n,t}^{(1y)}$ follows an AR(1) process, then we have[21]

$$\mu_{n,t}^{(1y)} \; = \overline{\mu}_n \; + \; H \cdot \frac{1-\phi_n}{1-\phi_n^H} \cdot (\mu_{n,t} - \overline{\mu}_n) \tag{18}$$

where $\overline{\mu}_n$ is the unconditional mean of $\mu_{n,t}$, H is the $\mu_{n,t}$ forecasting horizon, and ϕ_n is the $\mu_{n,t}$ AR(1) persistence. We then set $H = 10$ years (the most common horizon for our institution-year observations), estimate $\overline{\mu}_n$ from $1/T_n \cdot \sum_{t=1}^{T_n} \mu_{n,t}$, and estimate ϕ_n from the sample persistence of $\mu_{n,t}$.

Table 7 (Panel A, second block) estimates Equation 16 using our $\mu_{n,t}^{(1y)}$ proxy from Equation 18. The results are striking. First, in the pooled estimation, we find that $a = 0.00$ ($t_{stat} = 0.15$) and $b = 1.04$ ($t_{stat} = 2.82$) and are not able to reject the joint hypothesis that $a = 0$ and $b = 1$. Moreover, regardless of whether we focus on variation across asset classes or over time, the b coefficient remains statistically indistinguishable from 1. The R^2 values are also non-trivial. For instance, the pooled R^2 is 7.1% and the within R^2 values are higher than 5%. To understand whether belief variation (across asset classes and over time) improves upon belief homogeneity, we also create $R^2_{a=0,b=1}$, which reflects an R^2 measure that evaluates

[21] Specifically, if we have $(\mu_{n,t+1}^{(1y)} - \overline{\mu}_n) = \phi_n \cdot (\mu_{n,t}^{(1y)} - \overline{\mu}_n) + \epsilon_{t+1}$, then:

$$\mu_{n,t} - \overline{\mu}_n \; = \frac{1}{H} \cdot \sum_{h=0}^{H-1} \mathbb{E}_t[\mu_{n,t+h}^{(1y)}] \; = \; \frac{1}{H} \cdot \sum_{h=0}^{H-1} \phi_n^h \cdot (\mu_{n,t}^{(1y)} - \overline{\mu}_n) \; = \; \frac{1}{H} \cdot \frac{1-\phi_n^H}{1-\phi_n} \cdot (\mu_{n,t}^{(1y)} - \overline{\mu}_n)$$

which yields Equation 18. Note that the unconditional expectation of $\mu_{n,t}$ and the persistency of $\mu_{n,t} - \overline{\mu}_n$ are the same as the unconditional expectation of $\mu_{n,t}^{(1y)}$ and persistency of $\mu_{n,t}^{(1y)} - \overline{\mu}_n^{(1y)}$, which allows us to estimate $\overline{\mu}_n$ and ϕ_n directly from $\mu_{n,t}$ data (as described below Equation 18).

the predictability of $\mu_{n,t}^{(1y)}$ against constant belief benchmarks (and can be negative).[22] The $R^2_{a=0,b=1}$ values suggest that belief variation improves predictability both across asset classes and over time. Finally, we also consider an out-of-sample R^2 metric (R^2_{OOS}), which evaluates the predictability of $\mu_{n,t}^{(1y)}$ against benchmark models based on historical average returns (and can also be negative).[23] R^2_{OOS} is strong both across asset classes (12.6%) and over time (12.9%).[24]

Table 7 (Panel A, third block) considers a regression analogous to Equation 16, but that uses 3-year subjective expected returns ($\mu_{n,t}^{(3y)}$) to predict the average excess return over

[22]Mathematically, we calculate

$$R^2_{a=0,b=1} = 1 - \frac{\sum_{n=1}^{N}\sum_{t=1}^{T_n}(r_{n,t+1} - \mu_{n,t}^{(1y)})^2}{\sum_{n=1}^{N}\sum_{t=1}^{T_n}(r_{n,t+1} - \overline{\mu}_{n,t}^{(1y)})^2}$$

where $\overline{\mu}_{n,t}^{(1y)} = \overline{\mu}^{(1y)} = \sum_{n=1}^{N}\sum_{t=1}^{T_n}\mu_{n,t}^{(1y)}$ when considering all data variation, $\overline{\mu}_{n,t}^{(1y)} = \overline{\mu}_{t}^{(1y)} = \sum_{n=1}^{N}\mu_{n,t}^{(1y)}$ when considering variation across asset classes, and $\overline{\mu}_{n,t}^{(1y)} = \overline{\mu}_{n}^{(1y)} = \sum_{t=1}^{T_n}\mu_{n,t}^{(1y)}$ when considering variation over time. $R^2_{a=0,b=1}$ is defined analogously in columns that use alternative independent and/or dependent variables in the regression.

[23]Mathematically, we calculate

$$R^2_{OOS} = 1 - \frac{\sum_{n=1}^{N}\sum_{t=1}^{T_n}(r_{n,t+1} - \mu_{n,t}^{(1y)})^2}{\sum_{n=1}^{N}\sum_{t=1}^{T_n}(r_{n,t+1} - \widehat{\mu}_{n,t})^2}$$

where the definition of $\widehat{\mu}_{n,t}$ depends on whether we are considering variation across asset classes or over time. In both cases, we start by defining $\overline{r}_{n,t}$ to be the historical (expanding window) average return for asset class n up to time t. To ensure reasonable historical averages, we require at least 20 years in the expanding window, setting to missing any $\overline{r}_{n,t}$ value that does not satisfy this criterium. Then, in the case of variation over time, we use $\widehat{\mu}_{n,t} = \overline{r}_{n,t}$ so that forecasting errors for asset class-year observations with missing $\overline{r}_{n,t}$ are not considered (in both the numerator and denominator of R^2_{OOS}). In the case of variation across asset classes, we use $\widehat{\mu}_{n,t} = \widehat{\mu}_t = \sum_{n=1}^{N_t}\overline{r}_{n,t}$, where N_t reflects the number of asset classes in the given cross-section. R^2_{OOS} is defined analogously in columns that use alternative independent and/or dependent variables in the regression.

[24]One worry with these predictability results is that some institutions have CMA reports dated as of the beginning of year $t+1$. In principle, these reports are based on information institutions had as of the end of year t, and thus are still valid for predicting year $t+1$ returns. However, as a robustness check, we forecast annual returns from the beginning of Q2 of year $t+1$ to the end of Q1 of year $t+2$ and find results that are similar to our baseline results in Table 7 (see Panel A of Internet Appendix Table IA.2). Another potential worry is that our aggregation method is not "out-of-sample" as it requires full sample estimates of institution fixed effects (to deal with variation in the composition of institutions in our sample over time). As another robustness check, we forecast returns using $\frac{1}{J_t}\sum_{j=1}^{J_t}\mu_{j,n,t}$, which requires no future information. The results are similar to our baseline results in Table 7 (see Panel B of Internet Appendix Table IA.2).

the next three years ($\overline{r}^{(3y)}_{n,t\to t+3}$).[25] The idea is to evaluate the predictability of longer-term returns given that beliefs have multi-year horizons. However, our sample is short on the time dimension so that it is not meaningful to evaluate predictability at horizons that are too long (e.g., with a 3-year horizon, the longest time series in our sample has less than 12 independent 3-year return observations). The results indicate that $\mu^{(3y)}_{n,t}$ predicts $\overline{r}^{(3y)}_{n,t\to t+3}$ with similar strength as $\mu^{(1y)}_{n,t}$ predicts $\overline{r}^{(1y)}_{n,t+1}$ (somewhat better in the cross-section and somewhat worse in the time-series). Importantly, we continue to not reject the $a = 0$ and $b = 1$ implications from rational expectations.

Table 7 (Panel B, first two blocks) repeats the predictability exercise for risk measures (but without any adjustment for horizon).[26] In both cases ($\sigma^2_{n,t}$ and $\beta^{(e)}_{n,t}$), we observe strong predictability across asset classes and b coefficients with year fixed effects that are not that far from 1 (albeit we can statistically reject the $b = 1$ restriction for $\sigma^2_{n,t}$). However, we observe no predictability over time and the b coefficient with asset class fixed effects is negative (and statistically insignificant) in both cases. Results for $\beta^{(p)}_{n,t}$ are similar to the results for $\beta^{(e)}_{n,t}$, and thus we do not tabulate them to conserve space. So, our institutions seem to reasonably predict risk across assets. However, they are unable to predict risk variation over time. One limitation of this analysis is that if short-term risk and long-term risk are disconnected, then it is possible that time variation in subjective risk predicts time variation in long-term realized risk. We cannot meaningfully test this hypothesis given the short time series dimension of our sample.

Table 7 (Panel B, third block) repeats the predictability exercise for $\alpha^{(e)}_{n,t}$ with an adjustment for horizon analogous to the $\mu^{(1y)}_{n,t}$ adjustment (so, we call the belief quantity $\alpha^{(e,1y)}_{n,t}$). There is little connection between $\alpha^{(e,1y)}_{n,t}$ and subsequent alphas. While the b coefficient is positive in all specifications, it is always below 0.40 and statistically insignificant. Moreover, the R^2 values are all around 0.5%, which shows that subjective alphas explain almost no variation in subsequent realized alphas. Indeed, the $R^2_{a=0,b=1}$ values are all negative, indicating

[25] Specifically, we have $\overline{r}^{(3y)}_{n,t\to t+3} = \frac{1}{3}\cdot\sum_{h=1}^{3} r_{t+h}$ and $\mu^{(3y)}_{n,t} = \overline{\mu}_n + \frac{1}{3}\cdot\frac{1-\phi_n^H}{1-\phi_n}\cdot(\mu^{(1y)}_{n,t} - \overline{\mu}_n)$.

[26] Adjusting for horizon in expected risk is more challenging because the H-year subjective risk is not the subjective expectation of the average of 1-year realized risks over the next H-years.

that variation in beliefs is detrimental to forecasting alphas (i.e., removing subjective alpha heterogeneity improves alpha forecasts). We do see positive R^2_{OOS} values. However, given the negative $R^2_{a=0,b=1}$ values, this result is simply a reflection of the fact that historical alphas are an even worse predictor of future alphas than subjective alphas are.[27] Results for $\alpha^{(p,1y)}_{n,t}$ are similar to the results for $\alpha^{(e,1y)}_{n,t}$, and thus we do not tabulate them to conserve space. In a nutshell, the alpha evidence indicates that a Black CAPM type of model with all alphas equal to a single positive number (the Black CAPM intercept) provides a better forecasting model than the subjective alphas of our institutions do.

To further explore return predictability over time, Table 8 shows results for Equation 16 estimated separately for each asset class using $\mu^{(1y)}_{n,t}$.[28] Two key findings emerge. First, we can only reject the $b = 1$ hypothesis for *US High Yield Corp Bonds*. Second, R^2_{OOS} values are generally positive, with the only exception being *Hedge Funds* (with $R^2_{OOS} = -5.7\%$). Overall, the subjective expected returns for our institutions vary in a manner consistent with rational expectations.

5 Conclusion

This paper studies the link between the subjective risk and return expectations of institutional investors. Specifically, we explore the long-term Capital Market Assumptions of asset managers and institutional investor consultants from 1987 to 2022 to learn about their subjective views on the risks and returns of 19 assets classes. We uncover three stylized facts. First, there is a strong and positive subjective risk-return tradeoff, with most of the variability in subjective expected returns due to variability in subjective risk premia as opposed

[27]We also explore an R^2_{OOS} value that replaces the historical alphas with zero (the standard CAPM benchmark) and find that the R^2_{OOS} value remains positive (i.e., subjective alphas perform better than a zero alpha benchmark). This result is a consequence of the fact that the Black CAPM is a better (than the standard CAPM) description of the data (both the subjective beliefs data and the realized return data). Specifically, as discussed in Subsection 3.1, subjective alphas are positive on average, a pattern also observed in realized alphas (see, for example, Figures 11(e) and 12(e)).

[28]Since there is no risk or alpha predictability over time (as per Table 7), we do not provide time-series risk or alpha predictions by asset class in Table 8.

to subjective alphas. Second, belief variation and the positive risk-return tradeoff are both stronger across asset classes than across institutions. And third, the subjective expected returns of our institutions predict subsequent realized returns across asset classes and over time.

Our paper provides an important contribution to the literature on subjective beliefs. In particular, our findings imply that models for the subjective beliefs of institutional investors should reflect a positive risk-return tradeoff. Additionally, in our setting, accounting for this risk-return trade-off when modeling subjective beliefs across asset classes is even more important than incorporating average belief distortions or belief heterogeneity (despite the presence of both in the data to some degree).

Our results also open the door to many new questions, some of which we explore in simultaneous work. For instance, Couts et al. (2023) study the drivers of the market risk premium embedded in the subjective beliefs of institutional investors. Relatedly, Andonov et al. (2023) study the pass through of the overall beliefs of institutional investors to their portfolio allocations. We hope our work can shed light on these important outstanding issues in asset pricing.

References

Adam, K., A. Marcet, and J. Bautel (2017). "Stock Price Booms and Expected Capital Gains". In: *American Economic Review* 107.8, pp. 2352–2408.

Adam, K., D. Matveev, and S. Nagel (2021). "Do survey expectations of stock returns reflect risk adjustments?" In: *Journal of Monetary Economics* 117, pp. 723–740.

Adam, K. and S. Nagel (2023). "Expectations Data in Asset Pricing". In: *Handbook of Economic Expectations*. Academic Press. Chap. 16, pp. 477–506.

Amromin, G. and S. A. Sharpe (2014). "From the Horse's Mouth: Economic Conditions and Investor Expectations of Risk and Return". In: *Management Science* 60.4, pp. 845–866.

Andonov, A., S. J. Couts, A. S. Gonçalves, J. A. Loudis, and A. Rossi (2023). "Subjective Beliefs and the Portfolio Allocation of Institutional Investors". Working Paper.

Andonov, A. and J. D. Rauh (2022). “The Return Expectations of Public Pension Funds”. In: *Review of Financial Studies* 35.8, pp. 3777–3822.

Atmaz, A., H. Gulen, S. Cassella, and F. Ruan (2023). “Contrarians, Extrapolators, and Stock Market Momentum and Reversal”. In: *Management Science* Forthcoming.

Bacchetta, P., E. Mertens, and E. van Wincoop (2009). “Predictability in financial markets: What do survey expectations tell us?” In: *Journal of International Money and Finance* 28, pp. 406–426.

Bastianello, F. (2022). “Time-Series and Cross-Section of Risk Premia Expectations: A Bottom-Up Approach”. Working Paper.

Begenau, J., P. Liang, and E. Siriwardane (2023). “The Rise in Alternatives”. Working Paper.

Beutel, J. and M. Weber (2022). “Beliefs and Portfolios: Causal Evidence”. Working Paper.

Black, F. (1972). “Capital Market Equilibrium with Restricted Borrowing”. In: *Journal of Business* 45.3, pp. 444–455.

Bondt, W. F. D. (1993). “Betting on trends: Intuitive forecasts of financial risk and return”. In: *International Journal of Forecasting* 9, pp. 355–371.

Boons, M., G. Ottonello, and R. I. Valkanov (2023). “Excess Volatility in Professional Stock Return Forecasts”. Working Paper.

Bordalo, P., N. Gennaioli, R. L. Porta, and A. Shleifer (2019). “Diagnostic Expectations and Stock Returns”. In: *Journal of Finance* 74.6, pp. 2839–2874.

Bordalo, P., N. Gennaioli, R. L. Porta, and A. Shleifer (2022). “Belief Overreaction and Stock Market Puzzles”. Working Paper.

Brav, A., R. Lehavy, and R. Michaely (2005). “Using Expectations to Test Asset Pricing Models”. In: *Financial Management* 34.3, pp. 31–64.

Cassella, S. and H. Gulen (2018). “Extrapolation Bias and the Predictability of Stock Returns by Price-Scaled Variables”. In: *Review of Financial Studies* 31.11, pp. 4345–4397.

Cassella, S., B. Golez, H. Gulen, and P. Kelly (2022). “Horizon Bias and the Term Structure of Equity Returns”. In: *Review of Financial Studies* 36.3, pp. 1253–1288.

Chaudhry, A. (2022). “Do Subjective Growth Expectations Matter for Asset Prices?” Working Paper.

Chen, L., Z. Da, and X. Zhao (2013). “What Drives Stock Price Movements?” In: *Review of Financial Studies* 26.4, pp. 841–876.

Cieslak, A. (2018). “Short-Rate Expectations and Unexpected Returns in Treasury Bonds”. In: *Review of Financial Studies* 31.9, pp. 3265–3306.

Couts, S. J., A. S. Gonçalves, J. Loudis, and Y. Liu (2023). “The Subjective Risk Prices and Risk Quantities of Institutional Investors”. Working Paper.

Dahlquist, M. and M. Ibert (2023). “Equity Return Expectations and Portfolios: Evidence from Large Asset Managers”. Working Paper.

Dahlquist, M., M. Ibert, and F. Wilke (2023). “Are Subjective Expectations Formed as in Rational Expectations Models of Active Management?” Working Paper.

De La O, R., X. Han, and S. Myers (2022). “The Cross-section of Subjective Expectations: Understanding Prices and Anomalies”. Working Paper.

De La O, R. and S. Myers (2021). “Subjective Cash Flow Expectations and Discount Rate Expectations”. In: *Journal of Finance* 76.3, pp. 1339–1387.

Egan, M., A. MacKay, and H. Yang (2023). “What Drives Variation in Investor Portfolios? Estimating the Roles of Beliefs and Risk Preferences”. Working Paper.

Fama, E. F. and K. R. French (2002). “The Equity Premium”. In: *Journal of Finance* 57.2, pp. 637–659.

Fama, E. F. and J. D. MacBeth (1973). “Risk, Return and Equilibrium: Empirical Tests”. In: *Journal of Political Economy* 81.3, pp. 607–636.

Frazzini, A. and L. H. Pedersen (2014). “Betting Against Beta”. In: *Journal of Financial Economics* 111.1, pp. 1–25.

Froot, K. A. (1989). “New Hope for the Expectations Hypothesis of the Term Structure of Interest Rates”. In: *Journal of Finance* 44.2, pp. 283–305.

Gandhi, M., N. J. Gormsen, and E. Lazarus (2022). “Does the Market Understand Time Variation in the Equity Premium?” Working Paper.

Giglio, S., M. Maggiori, J. Stroebel, and S. Utkus (2021). "Five Facts About Beliefs and Portfolios". In: *American Economic Review* 111.5, pp. 1481–1522.

Gnan, P. and M. Schleritzko (2023). "Revisiting Discount Rates: New Evidence from Surveys". Working Paper.

Gormsen, N. J. and K. Huber (2022). "Corporate Discount Rates". Working Paper.

Gormsen, N. J. and K. Huber (2023). "Equity Factors and Firms' Perceived Cost of Capital". Working Paper.

Greenwood, R. and A. Shleifer (2014). "Expectations of Returns and Expected Returns". In: *Review of Financial Studies* 27, pp. 714–746.

Jensen, T. I. (2022). "Subjective Risk and Return". Working Paper.

Jo, C., C. Lin, and Y. You (2022). "Subjective Risk-Return Trade-off". Working Paper.

Laudenbach, C., A. Weber, R. Weber, and J. Wohlfart (2023). "Beliefs About the Stock Market and Investment Choices: Evidence from a Field Experiment". Working Paper.

Li, F. (2022). "Retail Trading and Asset Prices: The Role of Changing Social Dynamics". Working Paper.

Lintner, J. (1965). "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets". In: *Review of Economics and Statistics* 47.1, pp. 13–37.

Lochstoer, L. A. and T. Muir (2022). "Volatility Expectations and Returns". In: *Journal of Finance* 77.2, pp. 1055–1096.

Loudis, J. A. (2022). "Stock Price Reactions to the Information and Bias in Analyst-Expected Returns". Working Paper.

Malmendier, U. and S. Nagel (2011). "Depression Babies: Do Macroeconomic Experiences Affect Risk Taking?" In: *Quarterly Journal of Economics* 126.1, pp. 373–416.

Malmendier, U. and S. Nagel (2016). "Learning from Inflation Experiences". In: *Quarterly Journal of Economics* 131.1, pp. 53–87.

Malmendier, U., S. Nagel, and Z. Yan (2021). "The making of hawks and doves". In: *Journal of Monetary Economics* 117, pp. 19–42.

Merton, R. C. (1980). "On Estimating the Expected Return on the Market: An Exploratory Investigation". In: *Journal of Financial Economics* 8, pp. 323–361.

Nagel, S. and Z. Xu (2021). "Asset Pricing with Fading Memory". In: *Review of Financial Studies* 35.5, pp. 2190–2245.

Nagel, S. and Z. Xu (2022). "Dynamics of Subjective Risk Premia". Working Paper.

Piazzesi, M., J. Salomao, and M. Schneider (2015). "Trend and Cycle in Bond Premia". Working Paper.

Roll, R. (1977). "A Critique of the Asset Pricing Theory's Tests". In: *Journal of Financial Economics* 4, pp. 129–176.

Sharpe, W. F. (1964). "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk". In: *Journal of Finance* 19.3, pp. 425–442.

Vissing-Jørgensen, A. (2003). "Perspectives on Behavioral Finance: Does "Irrationality" Disappear with Wealth? Evidence from Expectations and Actions". In: *NBER Macroeconomics Annual* 18.138-194, pp. 138–194.

Wang, R. (2021). "Subjective Return Expectations". Working Paper.

Wu, L. (2018). "Estimating risk-return relations with analysts price targets". In: *Journal of Banking and Finance* 93, pp. 183–197.

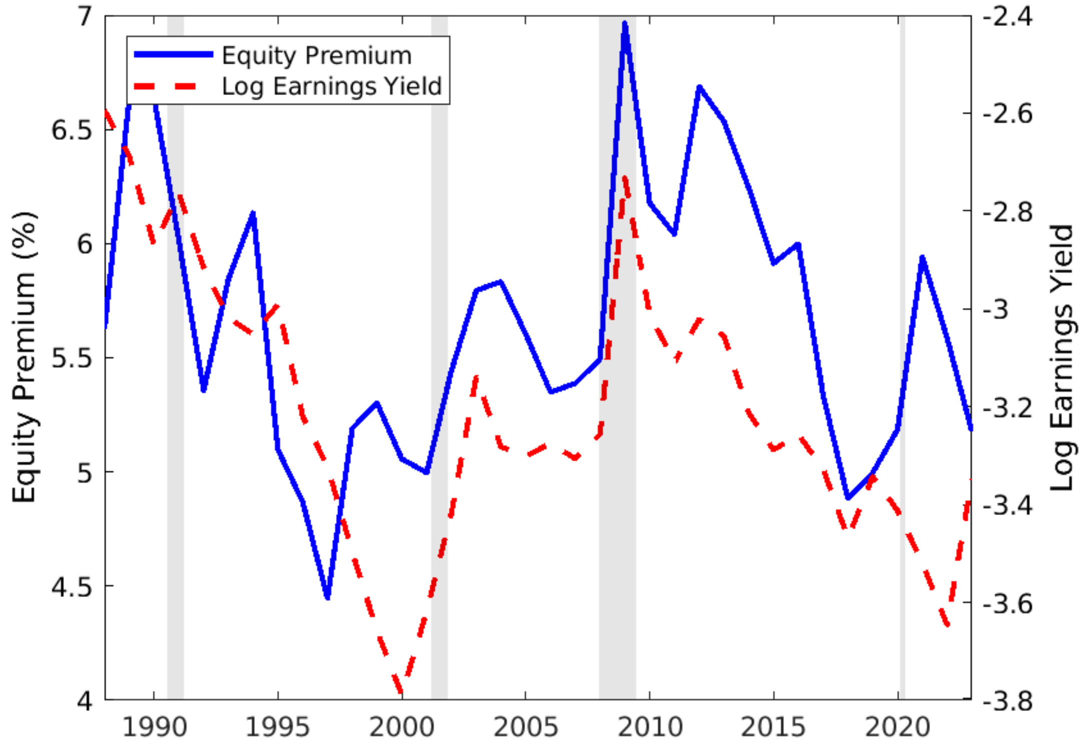


Figure 2
The Aggregate Subjective Equity Premium from Institutions

This figure plots the subjective equity premium aggregated across our institutions together with the S&P 500 earnings yield, proxied with $log(1/CAPE)$, where CAPE refers to the Cyclically Adjusted PE Ratio from Robert Shiller (available under http://www.econ.yale.edu/~shiller/data.htm). The shaded regions reflect US economic recessions as defined by the National Bureau of Economic Research (NBER). The subjective equity premium of institution j at time t is given by $\mu_{j,e,t} = \mathbb{E}_{j,t}[R_e] - \mathbb{E}_{j,t}[R_f]$ with R_e reflecting returns from *US Equities (Large Cap)* and R_f reflecting returns from *US Cash*. Section 2 provides more details about our subjective beliefs data and the analysis in this figure. Our aggregation method accounts for differences in sample composition over the years (see Subsection 4.1 for details). Specifically, we estimate a panel regression of $\mu_{j,e,t}$ onto institution fixed effects and year fixed effects (with no intercept and one manager dummy suppressed),

$$\mu_{j,e,t} = \overline{\mu}_{e,j} + \overline{\mu}_{e,t} + \eta_{j,e,t}$$

and then plot the following quantity:

$$\mu_{e,t} = \left[\frac{1}{T} \cdot \sum_{t=1}^{T} \left(\frac{1}{J_t} \sum_{j=1}^{J_t} \mu_{j,e,t}\right)\right] + \left(\overline{\mu}_{e,t} - \frac{1}{T} \cdot \sum_{t=1}^{T} \overline{\mu}_{e,t}\right)$$

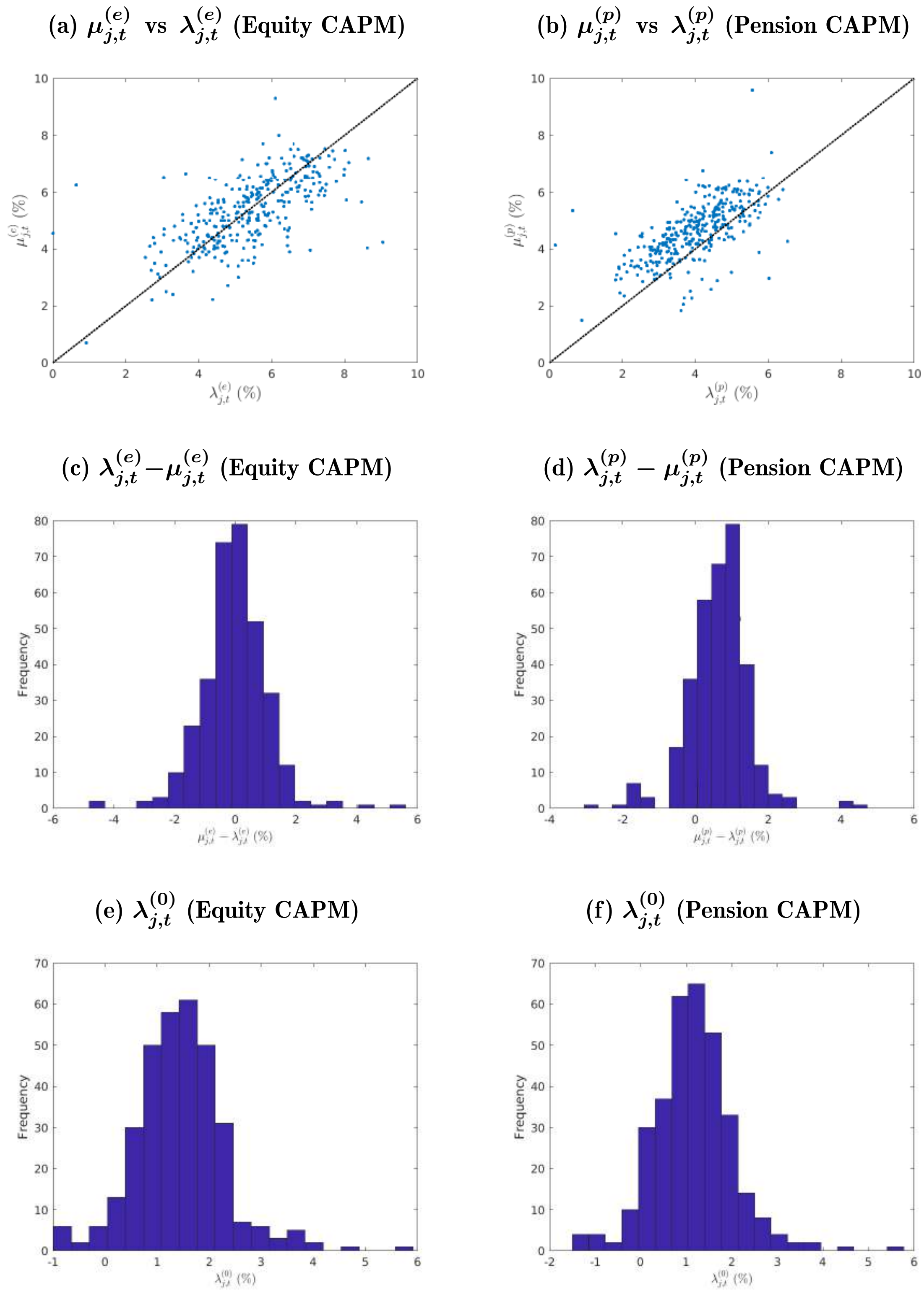


Figure 3

Consistency Between Subjective Expected Returns and the Pricing of Subjective βs

The pricing of subjective betas is based on the institution-year regression $\mu_{j,n,t} = \lambda^{(0)}_{j,t} + \lambda^{(m)}_{j,t} \cdot \beta^{(m)}_{j,n,t} + a^{(m)}_{j,n,t}$, where $\mu_{j,n,t}$ and $\beta^{(m)}_{j,n,t}$ are obtained directly from the beliefs of institution j at time t. Panels (a) to (d) display in different ways the distribution (across institution-year observations) of the subjective market expected return ($\mu^{(m)}_{j,t}$) against $\lambda^{(m)}_{j,t}$ while Panels (e) and (f) display the distribution of $\lambda^{(0)}_{j,t}$. We consider two models to determine βs. The first is the Equity CAPM ($m = e$), with market portfolio based on *US Equities (Large Cap)*. The second is the Pension CAPM ($m = p$), with market portfolio based on the aggregate allocation of US Public Pension Funds. Section 2.2 provides more details about our subjective beliefs data and Section 3.1 provides more details about the analysis reported in this figure.

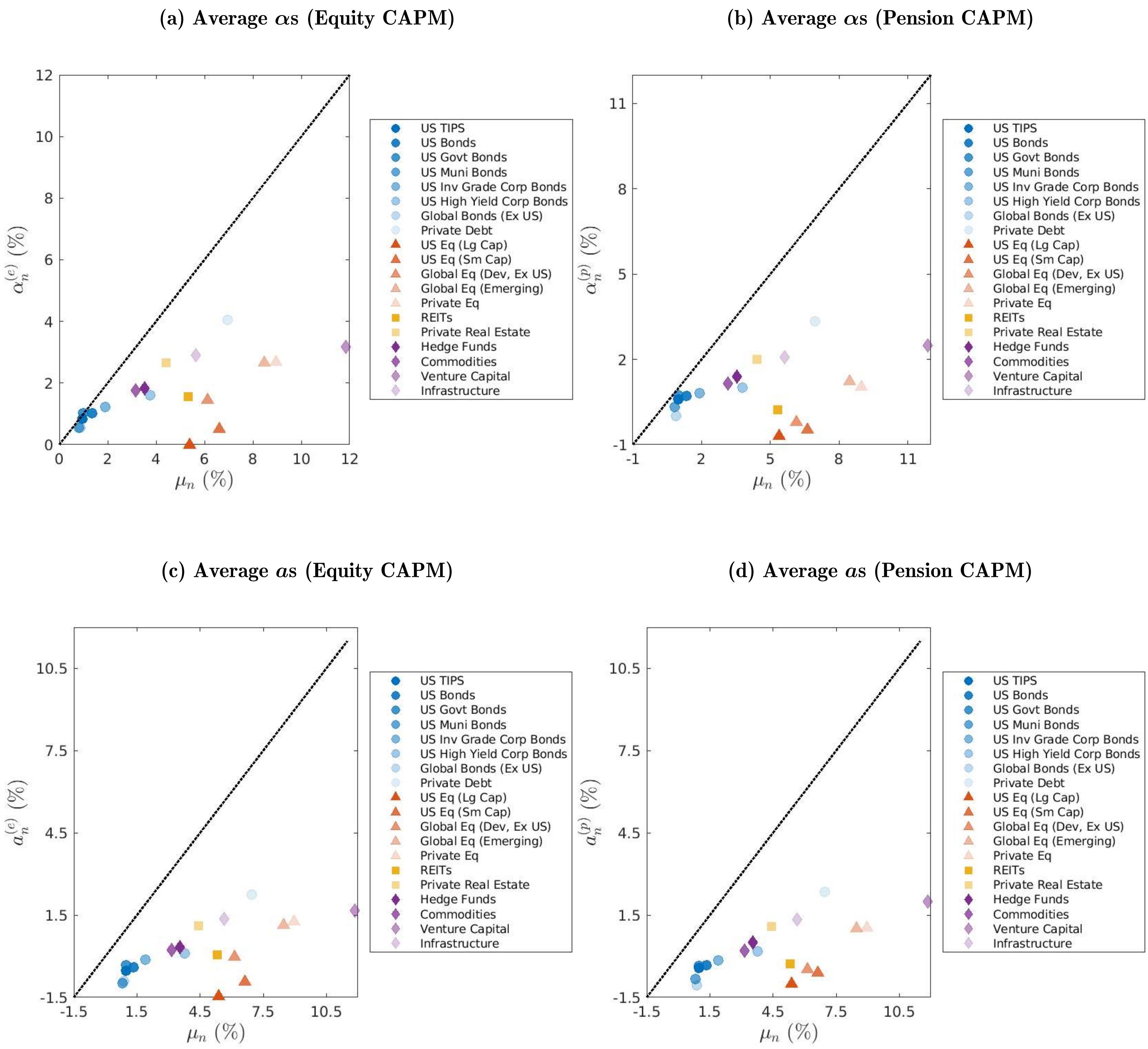


Figure 4

Average Subjective Pricing Errors for Each Asset Class

This figure plots the expected return of each asset class (averaged across institutions and years) against its pricing error (averaged across institutions and years). Pricing errors are based on Equation 6 ($\alpha^{(m)}$ in panels (a) and (b)) and Equation 5 ($a^{(m)}$ in panels (c) and (d)). We consider two models to determine βs. The first is the Equity CAPM ($m = e$), with market portfolio based on *US Equities (Large Cap)*. The second is the Pension CAPM ($m = p$), with market portfolio based on the aggregate allocation of US Public Pension Funds. Section 2.2 provides more details about our subjective beliefs data and Section 3.1 provides more details about the analysis reported in this figure.

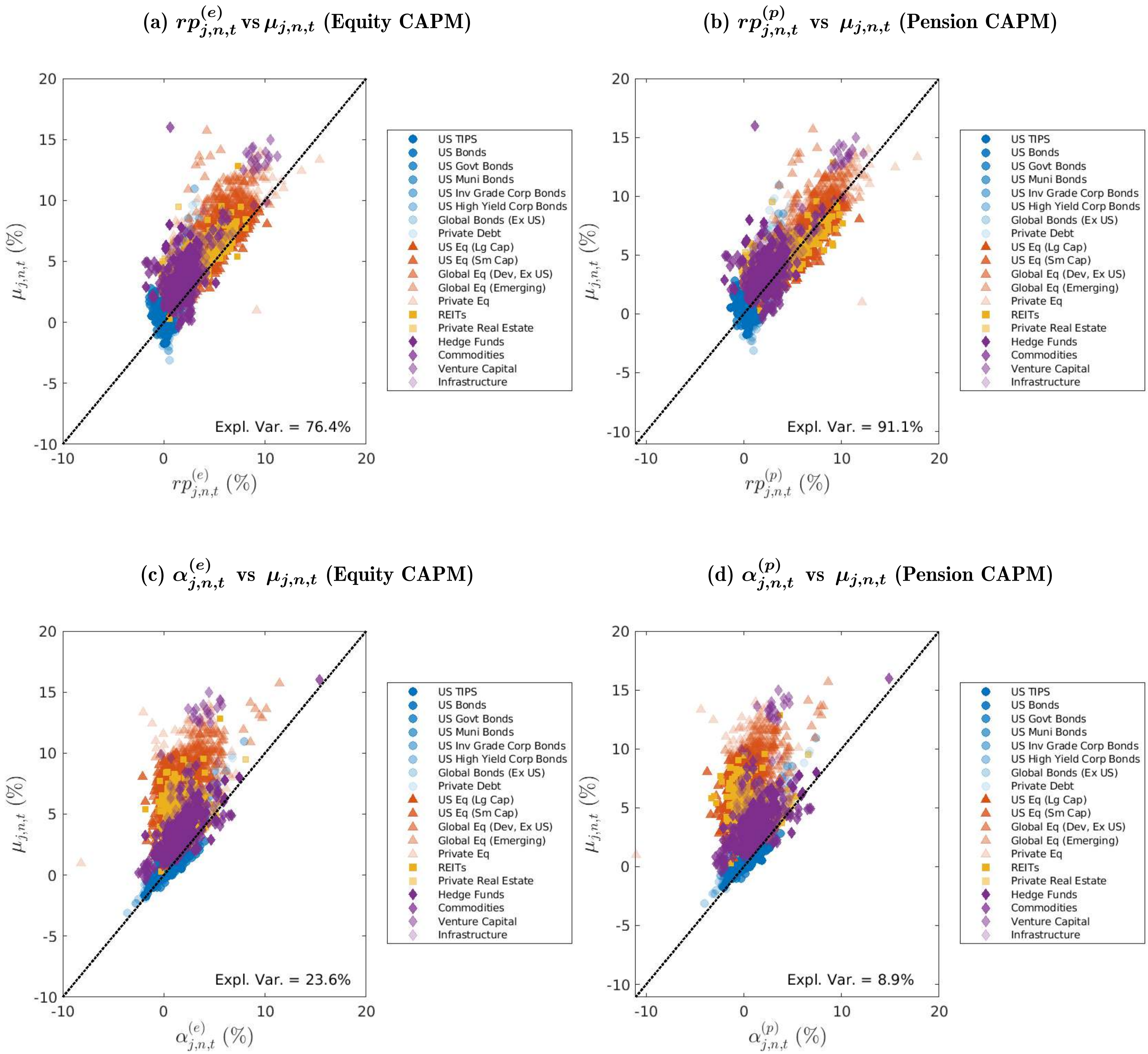


Figure 5

Subjective Expected Returns: Risk Premia vs Alphas

This figure plots the subjective expected return of each observation ($\mu_{j,n,t}$) against the respective subjective risk premium ($rp^{(m)}_{j,n,t} = \beta^{(m)}_{j,n,t} \cdot \mu^{(m)}_{j,t}$) or the respective subjective pricing error ($\alpha^{(m)}_{j,n,t}$) based on Equation 6. We consider two models to determine βs. The first is the Equity CAPM ($m = e$), with market portfolio based on *US Equities (Large Cap)*. The second is the Pension CAPM ($m = p$), with market portfolio based on the aggregate allocation of US Public Pension Funds. Section 2.2 provides more details about our subjective beliefs data and Section 3.1 provides more details about the analysis reported in this figure.

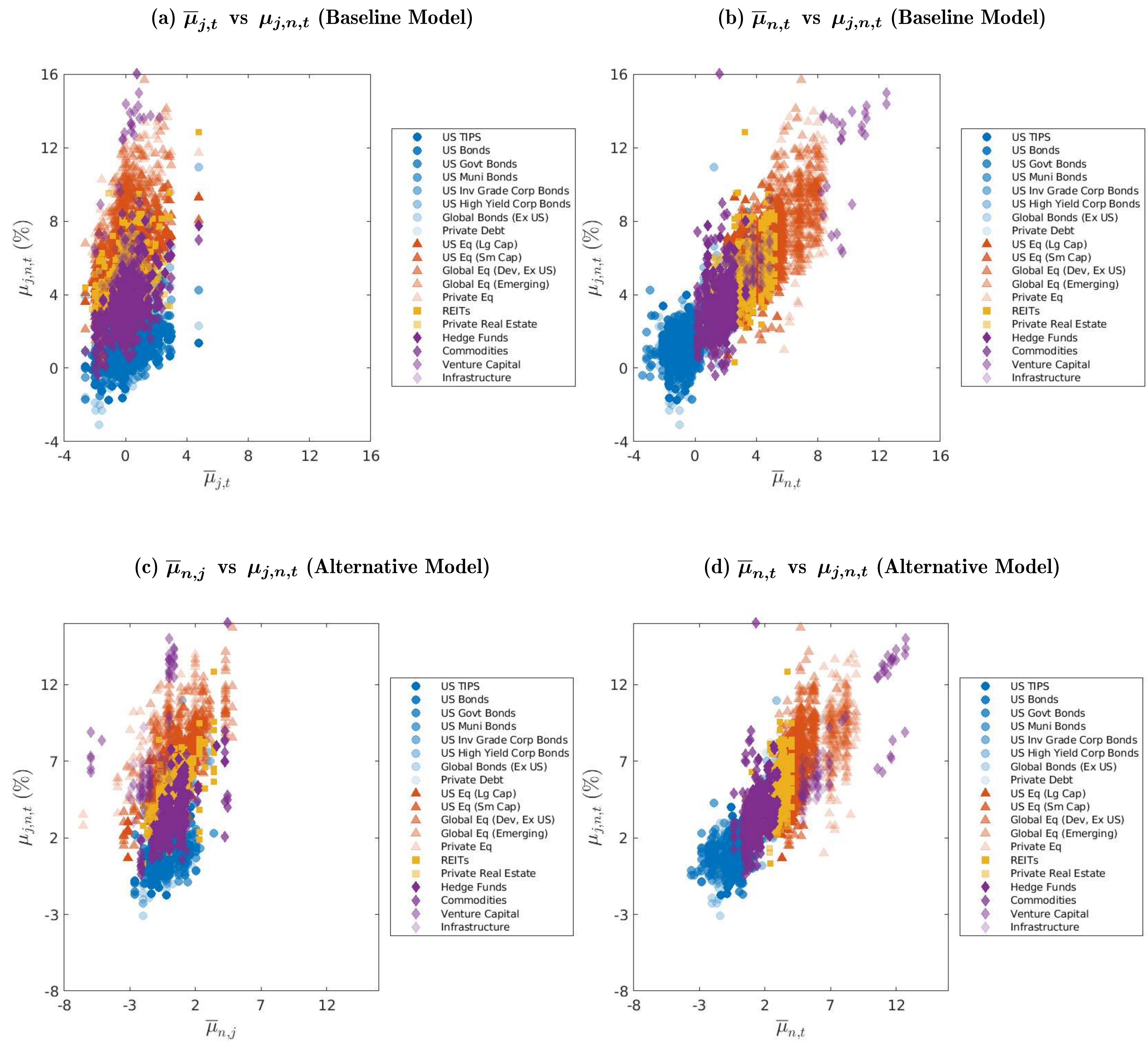
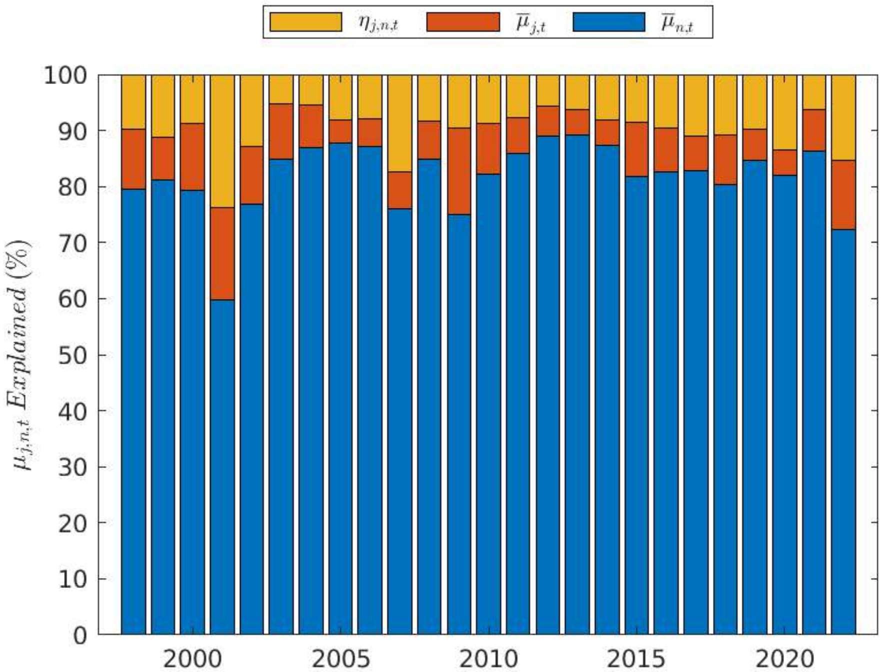


Figure 6

Heterogeneity in Subjective Expected Returns: Institution vs Asset Class Effects

This figure plots subjective expected returns ($\mu_{j,n,t}$) against institution or asset class fixed effects. The institution fixed effect is given by $\overline{\mu}_{j,t}$ or $\overline{\mu}_{n,j}$ depending on whether we are in the baseline model (Equation 8) or in the alternative model (Equation 10). The asset class fixed effect is always given by $\overline{\mu}_{n,t}$. Section 2.2 provides more details about our subjective beliefs data and Section 3.2 provides more details about the analysis reported in this figure.

(a) Baseline Model: $\mu_{j,n,t} = \overline{\mu}_{n,t} + \overline{\mu}_{j,t} + \eta_{j,n,t}$



(b) Alternative Model: $\mu_{j,n,t} = \overline{\mu}_{n,t} + \overline{\mu}_{n,j} + \eta_{j,n,t}$

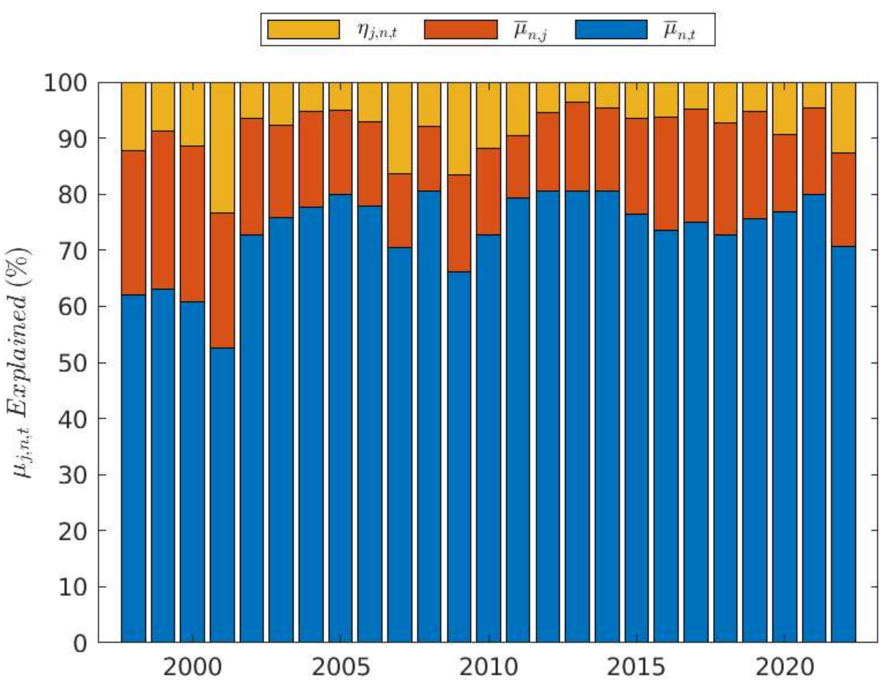


Figure 7

Decomposing Variability in Subjective Expected Returns: Institution vs Asset Class Effects

This figure plots the decomposition of within year expected return variability (based on Equations 9 and 11) into the effect of asset class heterogeneity (in blue), institution heterogeneity (in red), and residuals (in orange). The institution fixed effect is given by $\overline{\mu}_{j,t}$ or $\overline{\mu}_{n,j}$ depending on whether we are in the baseline model (Equation 8) or in the alternative model (Equation 10). The asset class fixed effect is always given by $\overline{\mu}_{n,t}$. Section 2.2 provides more details about our subjective beliefs data and Section 3.2 provides more details about the analysis reported in this figure.

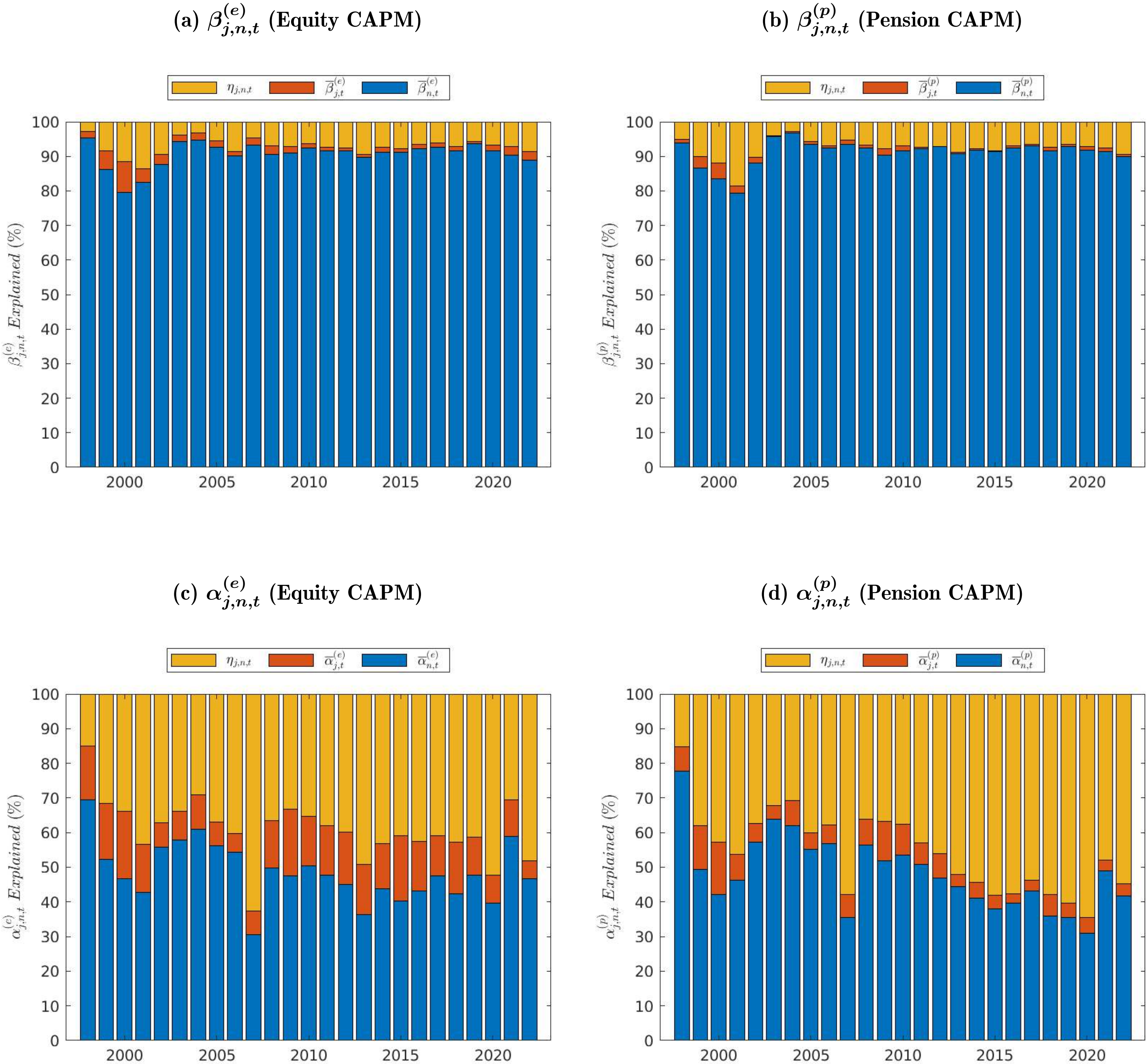


Figure 8

Decomposing Variability in Subjective βs and αs: Institution vs Asset Class Effects

This figure plots the decomposition of within year variability in βs and αs (based on Equation 13) into the effect of asset class heterogeneity (in blue), institution heterogeneity (in red), and residuals (in orange). The institution fixed effect is given by $\overline{\beta}_{j,t}$ or $\overline{\alpha}_{j,t}$ while the asset class fixed effect is given by $\overline{\beta}_{n,t}$ or $\overline{\alpha}_{n,t}$. We consider two models to determine βs. The first is the Equity CAPM ($m = e$), with market portfolio based on *US Equities (Large Cap)*. The second is the Pension CAPM ($m = p$), with market portfolio based on the aggregate allocation of US Public Pension Funds. Section 2.2 provides more details about our subjective beliefs data and Section 3.2 provides more details about the analysis reported in this figure.

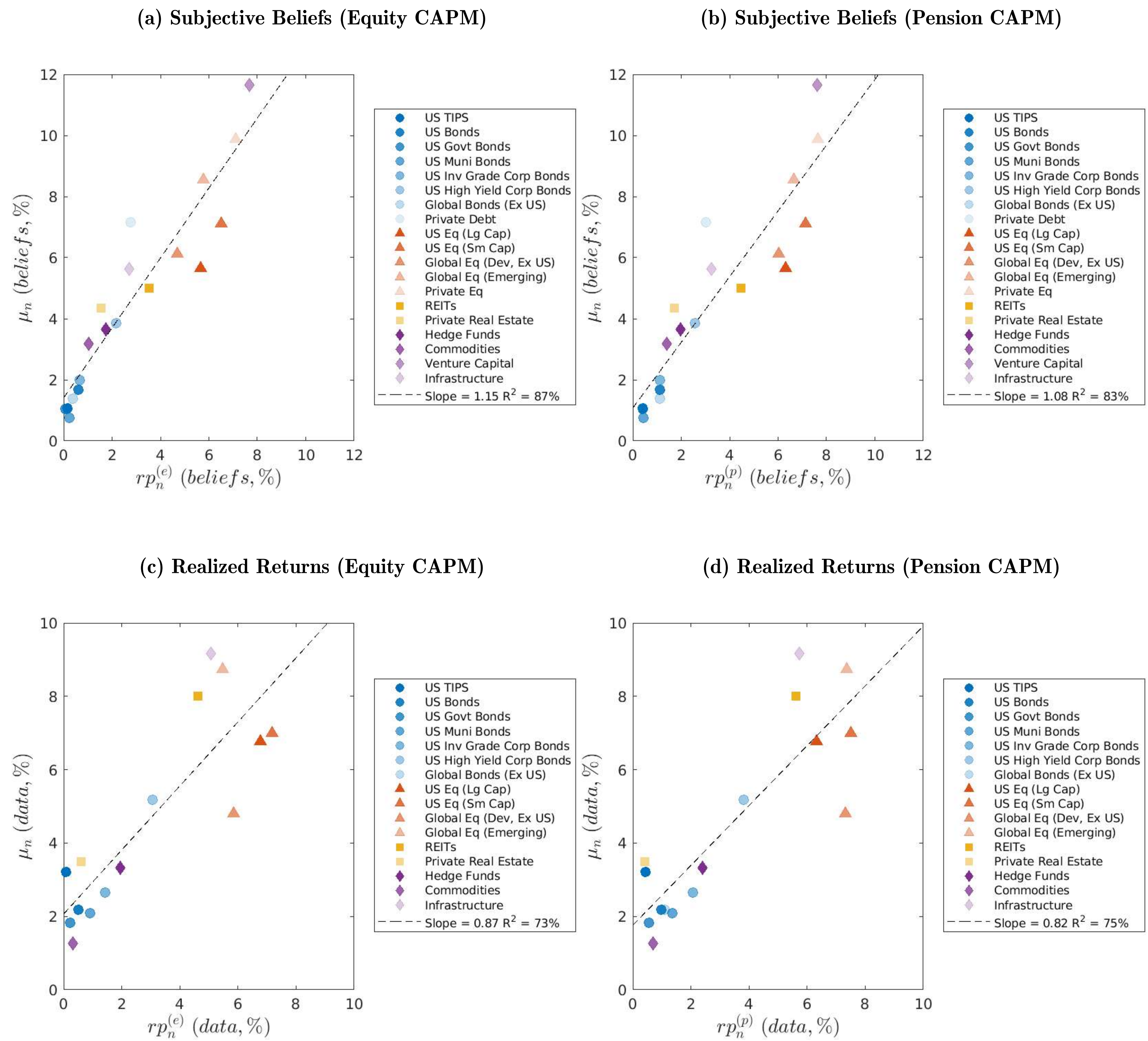


Figure 9

Risk-Return Tradeoff: Subjective Beliefs vs Realized Returns

This figure plots expected returns ($\mu_n = \mathbb{E}[r_n]$) against risk premia ($rp_n^{(m)} = \beta_n^{(m)} \cdot \mu^{(m)}$) across asset classes. Panels (a) and (b) use beliefs (averaged across institutions and years), and thus reflect the subjective risk-return tradeoff. Panels (c) and (d) use unconditional data moments (μ_n is based on average excess returns and $rp_n^{(m)}$ is based on estimated beta and market average excess return), and thus reflect the objective risk-return tradeoff. We consider two models to determine βs. The first is the Equity CAPM ($m = e$), with market portfolio based on *US Equities (Large Cap)*. The second is the Pension CAPM ($m = p$), with market portfolio based on the aggregate allocation of US Public Pension Funds. Sections 2.2 and 4.1 provide more details about the data and Section 4.2 provides more details about the analysis reported in this figure.

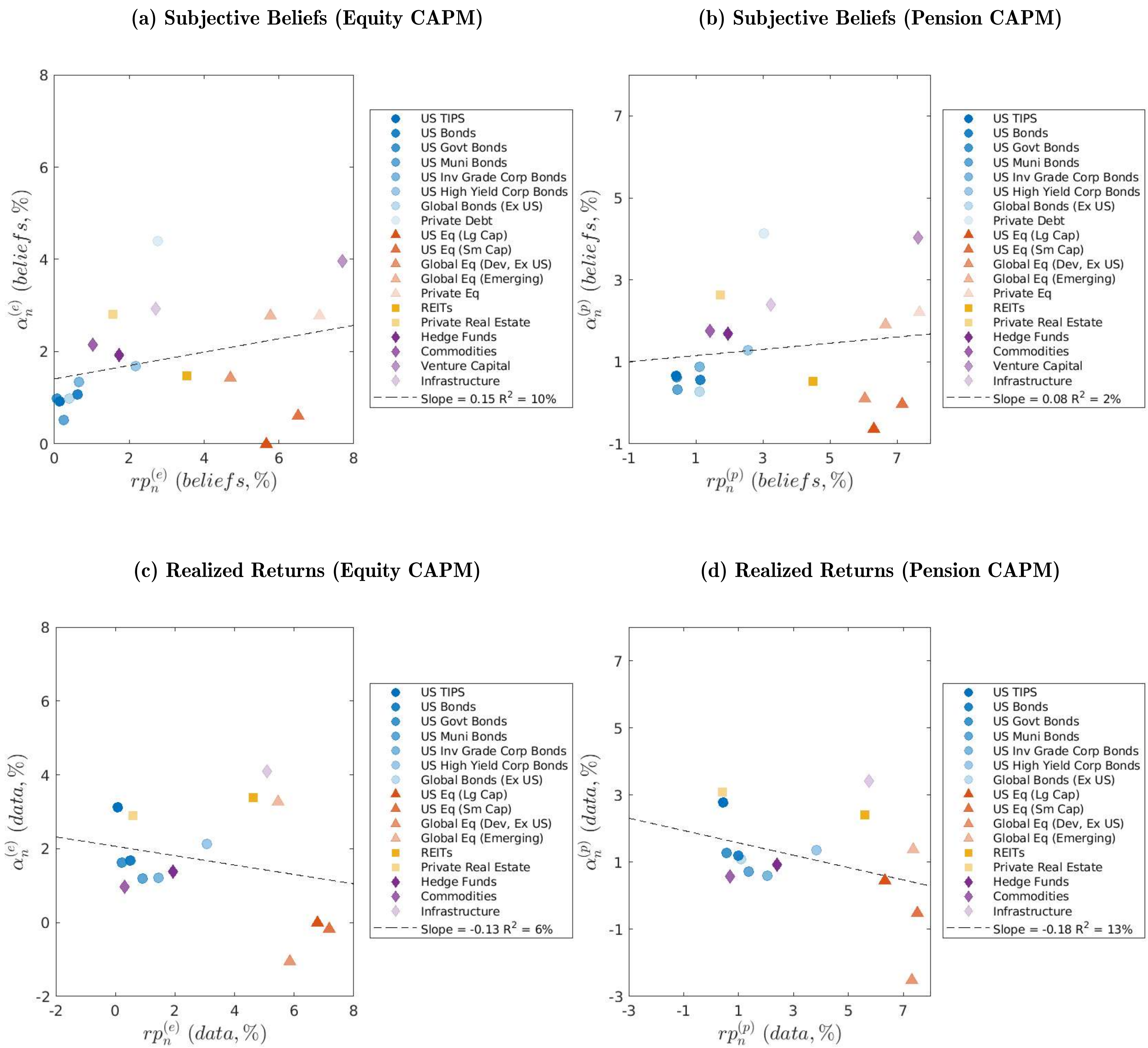
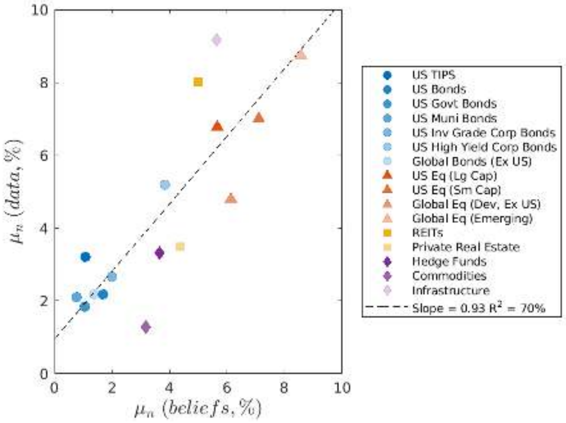


Figure 10

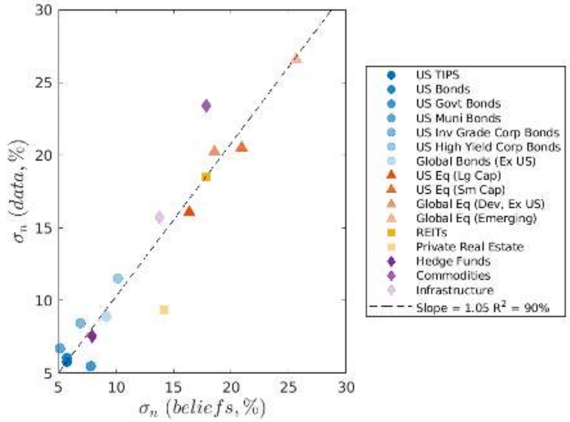
Low-Beta Anomaly: Subjective Beliefs vs Realized Returns

This figure plots alphas ($\alpha_n = \mu_n - rp_n^{(m)}$) against risk premia ($rp_n^{(m)} = \beta_n^{(m)} \cdot \mu^{(m)}$) across asset classes. Risk premia only vary with beta across asset classes (since the market risk premia is constant across asset classes). Panels (a) and (b) use beliefs (averaged across institutions and years), and thus reflect the subjective low-beta anomaly (or lack thereof). Panels (c) and (d) use unconditional data moments, and thus reflect the objective low-beta anomaly. We consider two models to determine βs. The first is the Equity CAPM ($m = e$), with market portfolio based on *US Equities (Large Cap)*. The second is the Pension CAPM ($m = p$), with market portfolio based on the aggregate allocation of US Public Pension Funds. Sections 2.2 and 4.1 provide more details about the data and Section 4.2 provides more details about the analysis reported in this figure.

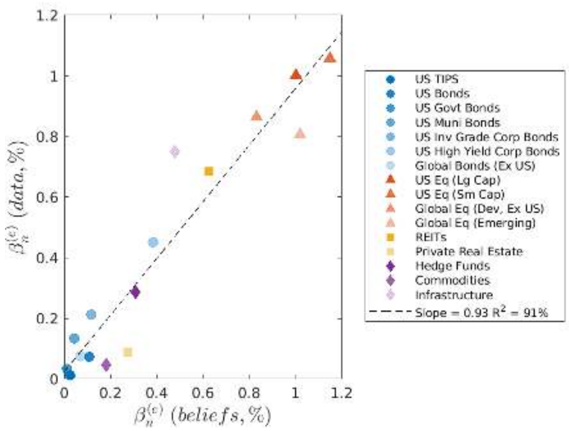
(a) μ_n



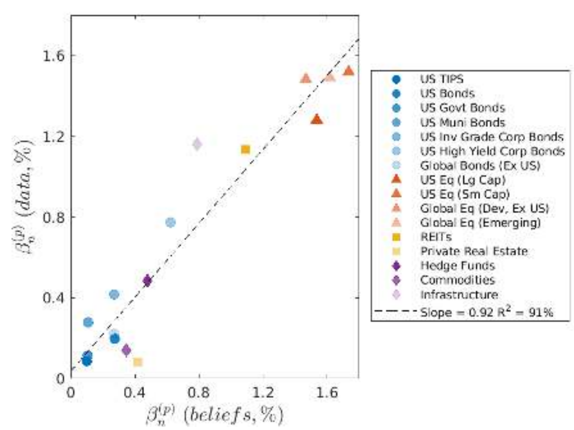
(b) σ_n



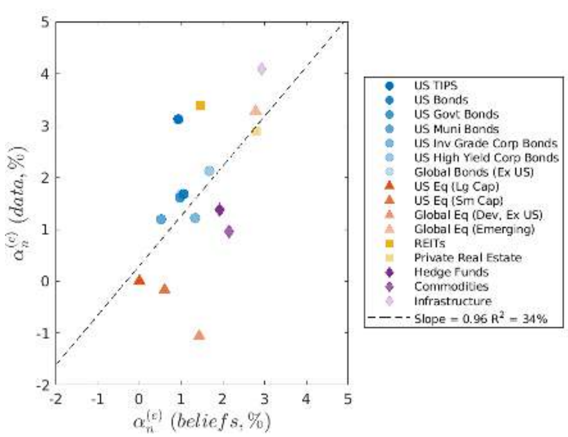
(c) $\beta_n^{(e)}$ (Equity CAPM)



(d) $\beta_n^{(p)}$ (Pension CAPM)



(e) $\alpha_n^{(e)}$ (Equity CAPM)



(f) $\alpha_n^{(p)}$ (Pension CAPM)

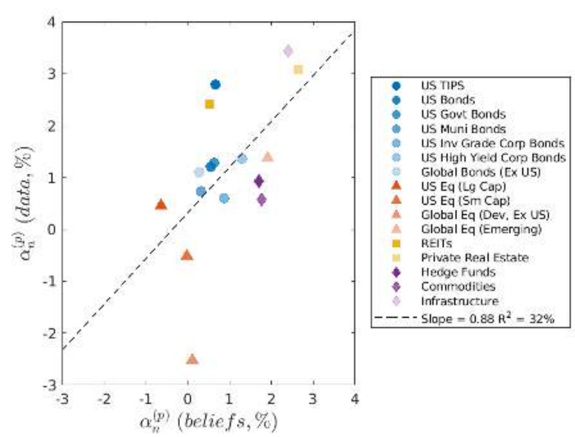


Figure 11

Consistency Between Subjective Beliefs and Realized Returns

This figure plots unconditional data moments against average values for their respective beliefs (averaged across institutions and years). We use returns over the full sample for each asset class (see dates in Table 6). For Panels (c) to (f), we consider two models to determine βs. The first is the Equity CAPM ($m = e$), with market portfolio based on *US Equities (Large Cap)*. The second is the Pension CAPM ($m = p$), with market portfolio based on the aggregate allocation of US Public Pension Funds. Sections 2.2 and 4.1 provide more details about the data and Section 4.3 provides more details about the analysis reported in this figure.

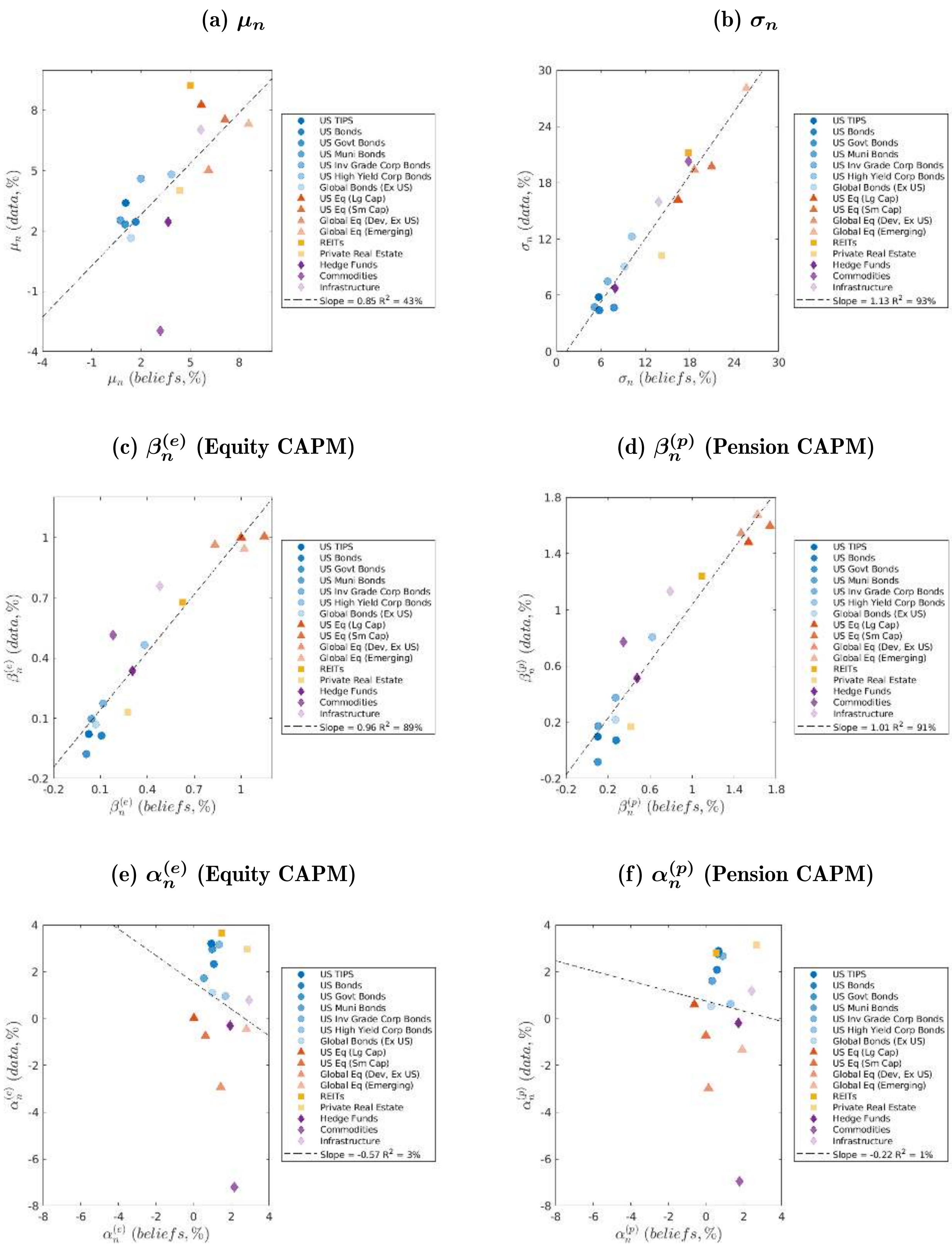


Figure 12

Consistency Between Subjective Beliefs and Realized Returns (Beliefs Matched to Subsequent Returns)

This figure plots unconditional data moments against average values for their respective beliefs (averaged across institutions and years). Beliefs are matched to subsequent year returns. For Panels (c) to (f), we consider two models to determine βs. The first is the Equity CAPM ($m = e$), with market portfolio based on *US Equities (Large Cap)*. The second is the Pension CAPM ($m = p$), with market portfolio based on the aggregate allocation of US Public Pension Funds. Sections 2.2 and 4.1 provide more details about the data and Section 4.3 provides more details about the analysis reported in this figure.

Table 1

Asset Managers and Institutional Investor Consultants in our Sample

The table reports information on the list of institutions that enter our sample at any point in time. Panel A details the asset managers (or simply "managers"), with information on their Assets Under Manager (AUM) ranking, dollar value, and fraction relative to the total AUM of the top 50 AUM managers in the world (data from the October 2022 report on the world's largest 500 asset managers by the Thinking Ahead Institute and the Pensions & Investments company). Panel B details the institutional investor consultants (or simply "consultants"), with information on their role as primary consultants for US public pension funds. Specifically, each row in Panel B reports information about the pension funds for which the given consultant is the primary consultant of on average from 2001 to 2021 (data from the Center for Retirement Research at Boston College). Section 2.1 provides more details about our subjective beliefs data. Institution names and the respective numbers will be provided after approval from the original data sources.

PANEL A - Asset Managers (or simply "Managers")

Manager Name	AUM Information		
	Rank	$ Value	% of Top 50
Barclays	-	-	-
BNY Mellon	9	2.43 Trillion	2.7%
Capital Group	7	2.72 Trillion	3.1%
Envestnet	> 50	0.36 Trillion	0.4%
Goldman Sachs	8	2.47 Trillion	2.8%
Invesco	15	1.61 Trillion	1.8%
JP Morgan	5	3.11 Trillion	3.5%
Morgan Stanley	18	1.49 Trillion	1.7%
Northern Trust	16	1.61 Trillion	1.8%
Nuveen	22	1.26 Trillion	1.4%
PFM	> 50	0.13 Trillion	0.2%
PIMCO	12	2.00 Trillion	2.3%
Russell Investments	> 50	0.34 Trillion	0.4%
SEI	> 50	0.30 Trillion	0.3%
T. Rowe Price	14	1.69 Trillion	1.9%
Voya	> 50	0.41 Trillion	0.5%
Wells Fargo	16	1.61 Trillion	1.8%
Asset Manager #18	-	-	-
Total	-	> 23.6 Trillion	> 26.6%

PANEL B - Institutional Investor Consultants (or simply "Consultants")

Consultant Name	Primary US Pension Fund Consultant of		
	# of Funds	% of Funds	% of Funds' AUM
Aon	19	8.8%	14.8%
Callan	24	11.2%	11.9%
CAPTRUST	-	-	-
Cliffwater	1	0.5%	0.6%
CSG	-	-	-
Meketa	6	2.6%	2.4%
Mercer	6	2.6%	2.5%
Milliman	1	0.7%	0.2%
NEPC	19	8.7%	4.5%
PCA	9	4.0%	7.4%
RVK	8	3.7%	6.2%
Sellwood	-	-	-
Strategic Inv Sol	1	0.3%	1.5%
Verus	4	1.7%	1.2%
Wilshire	11	5.3%	13.1%
WTW	2	0.7%	0.5%
Total	$\geq$ 111	$\geq$ 50.8%	$\geq$ 66.8%

Table 2
Sample Coverage (by Year)

This table reports information on our sample of Capital Market Assumptions (CMAs) over time. Panel A details, for each year, the number of asset managers (or simply "managers"), institutional investor consultants (or simply "consultants"), and institutions (managers+consultants) in our sample. We have data obtained directly from the CMAs of the underlying institutions (under "# of Institutions (direct data)") and data obtained indirectly from the reports of pension funds (which is given by "# of Institutions" - "# of Institutions (direct data)"). Panel A also reports, for each year, the number of unique asset classes in our sample as well as the average number of asset classes covered per institution. Panel B reports the number of institutions covering each of the asset classes in our sample. Section 2 provides more details about our subjective beliefs data.

PANEL A - Number of Managers, Consultants, and Asset Classes in our Sample (by Year)

	1987	1996	1997	1998	2000	2002	2004	2006	2008	2010	2012	2014	2016	2018	2020	2022
# of Institutions	1	1	3	4	5	8	7	10	11	14	17	14	16	20	21	24
# of Institutions (direct data)	0	1	3	4	5	5	5	7	8	10	12	12	13	17	20	22
# of Managers	0	0	1	1	1	1	1	1	4	5	6	5	6	10	11	15
# of Consultants	1	1	2	3	4	7	6	9	7	9	11	9	10	10	10	9
# of Asset Classes	4	7	13	13	13	16	16	18	18	19	20	20	20	20	20	20
Av # of Asset Classes per Institution	4	7	9	9	9	9	10	12	12	12	13	14	14	13	14	14

PANEL B - Number of Institutions Covering Each Asset Class in our Sample (by Year)

Asset Class	1987	1996	1997	1998	2000	2002	2004	2006	2008	2010	2012	2014	2016	2018	2020	2022
US Cash	1	1	3	4	5	8	7	10	11	14	17	14	16	20	21	24
US TIPS	0	0	0	1	1	5	6	9	10	14	16	13	14	17	17	21
US Bonds	1	1	3	4	5	8	7	10	11	12	16	13	16	17	17	19
US Govt Bonds	0	0	1	0	1	2	2	2	3	3	7	8	10	14	12	18
US Muni Bonds	0	0	0	0	0	1	1	1	3	4	6	6	6	8	8	10
US Inv Grade Corp Bonds	0	0	0	0	0	1	1	1	1	2	7	4	5	6	9	13
US High Yield Corp Bonds	0	0	2	3	5	5	5	7	9	11	13	11	13	16	17	21
Global Bonds (Ex US)	0	1	3	4	4	4	4	7	8	9	13	11	12	12	14	19
Private Debt	0	0	0	0	0	0	0	0	0	1	2	1	3	4	7	11
US Equities (Large Cap)	1	1	3	4	5	8	7	10	11	14	17	14	16	20	21	24
US Equities (Small Cap)	0	1	2	2	2	4	3	7	7	9	12	11	12	13	13	17
Global Equities (Dev, Ex US)	0	0	2	3	5	7	7	9	10	12	15	14	16	19	21	22
Global Equities (Emerging)	0	0	2	3	5	5	5	7	9	11	14	12	14	18	18	22
Private Equity	0	1	1	1	3	6	7	9	9	12	15	12	13	16	17	19
REITs	0	0	1	2	2	3	2	5	6	7	11	10	12	14	16	17
Private Real Estate	1	1	2	2	3	4	6	10	9	11	16	13	14	15	16	19
Hedge Funds	0	0	0	0	0	3	3	7	6	9	12	10	11	14	17	18
Commodities	0	0	1	1	0	0	0	6	7	10	13	11	13	17	17	19
Venture Capital	0	0	0	0	0	0	0	0	0	0	1	2	3	2	2	3
Infrastructure	0	0	0	0	0	0	0	1	2	2	4	4	4	6	10	10

Table 3
Average Subjective Beliefs (Pooled Across Institutions in 2022)

This table reports the average values for the belief quantities we observe at the end of 2022 pooled across institutions (we observe the same information in previous years, but for a subset of the asset classes available in 2022). Panel A reports average nominal returns, $\mathbb{E}[R]$, average volatilities, $\sigma[R]$, and average correlations. Panel B reports the same quantities, but for excess returns, $r = R - R_f$, with R_f proxied by the return on *US Cash*. Section 2.2 provides more details about our subjective beliefs data.

PANEL A - Raw Returns (R)

Asset Class	$\mathbb{E}[R]$	$\sigma[R]$	(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
(0) US Cash	3.0%	0.9%	1																			
(1) US TIPS	4.1%	6.0%	0.07	1																		
(2) US Bonds	4.5%	5.0%	0.15	0.74	1																	
(3) US Govt Bonds	4.1%	7.4%	0.16	0.65	0.84	1																
(4) US Muni Bonds	3.7%	4.9%	0.06	0.60	0.75	0.61	1															
(5) US Inv Grade Corp Bonds	5.5%	7.4%	0.03	0.66	0.84	0.60	0.71	1														
(6) US High Yield Corp Bonds	6.9%	9.8%	-0.05	0.35	0.30	-0.02	0.38	0.51	1													
(7) Global Bonds (Ex US)	3.8%	7.6%	0.09	0.59	0.68	0.62	0.54	0.66	0.31	1												
(8) Private Debt	8.7%	12.1%	-0.06	0.14	0.02	-0.21	0.10	0.35	0.67	0.14	1											
(9) US Equities (Large Cap)	7.6%	16.6%	-0.04	0.19	0.18	-0.09	0.17	0.35	0.69	0.19	0.59	1										
(10) US Equities (Small Cap)	8.9%	21.2%	-0.05	0.11	0.09	-0.18	0.13	0.32	0.67	0.12	0.58	0.89	1									
(11) Global Equities (Developed, Ex US)	8.5%	18.3%	-0.04	0.20	0.19	-0.09	0.18	0.36	0.67	0.29	0.55	0.83	0.78	1								
(12) Global Equities (Emerging)	10.4%	23.4%	-0.01	0.20	0.17	-0.12	0.17	0.33	0.64	0.19	0.51	0.72	0.69	0.80	1							
(13) Private Equity	10.8%	22.7%	-0.03	0.15	0.05	-0.17	0.10	0.29	0.61	0.16	0.62	0.76	0.73	0.70	0.63	1						
(14) REITs	8.0%	19.8%	-0.05	0.28	0.27	0.05	0.26	0.38	0.64	0.28	0.48	0.71	0.69	0.65	0.56	0.58	1					
(15) Private Real Estate	6.8%	13.6%	0.00	0.17	0.14	-0.02	0.10	0.17	0.38	0.11	0.43	0.44	0.44	0.38	0.34	0.49	0.61	1				
(16) Hedge Funds	6.3%	7.8%	0.00	0.20	0.13	-0.18	0.15	0.35	0.65	0.19	0.56	0.74	0.74	0.74	0.68	0.62	0.56	0.39	1			
(17) Commodities	5.6%	17.9%	-0.02	0.17	-0.06	-0.22	-0.05	0.11	0.35	0.06	0.34	0.33	0.33	0.40	0.40	0.33	0.28	0.21	0.43	1		
(18) Venture Capital	14.5%	29.5%	0.01	0.04	-0.09	-0.25	-0.04	0.19	0.59	-0.07	0.63	0.73	0.71	0.67	0.59	0.76	0.46	0.47	0.58	0.31	1	
(19) Infrastructure	8.2%	16.5%	-0.03	0.28	0.19	-0.09	0.21	0.32	0.62	0.26	0.58	0.70	0.66	0.70	0.63	0.66	0.64	0.51	0.61	0.40	0.55	1

PANEL B - Excess Returns ($r = R - R_f$)

Asset Class	$\mathbb{E}[r]$	$\sigma[r]$	(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
(0) US Cash	0.0%	0.0%	1																			
(1) US TIPS	1.1%	6.0%	0.00	1																		
(2) US Bonds	1.5%	5.0%	0.00	0.74	1																	
(3) US Govt Bonds	1.1%	7.3%	0.00	0.65	0.84	1																
(4) US Muni Bonds	0.6%	4.9%	0.00	0.60	0.75	0.61	1															
(5) US Inv Grade Corp Bonds	2.4%	7.4%	0.00	0.67	0.85	0.61	0.71	1														
(6) US High Yield Corp Bonds	3.9%	9.9%	0.00	0.36	0.31	0.00	0.39	0.52	1													
(7) Global Bonds (Ex US)	0.6%	7.6%	0.00	0.59	0.67	0.61	0.53	0.66	0.31	1												
(8) Private Debt	5.5%	12.2%	0.00	0.17	0.03	-0.19	0.11	0.36	0.67	0.15	1											
(9) US Equities (Large Cap)	4.6%	16.6%	0.00	0.20	0.20	-0.08	0.18	0.36	0.69	0.20	0.59	1										
(10) US Equities (Small Cap)	5.8%	21.3%	0.00	0.12	0.11	-0.17	0.14	0.32	0.67	0.12	0.58	0.89	1									
(11) Global Equities (Developed, Ex US)	5.4%	18.4%	0.00	0.21	0.20	-0.09	0.19	0.37	0.67	0.29	0.55	0.83	0.78	1								
(12) Global Equities (Emerging)	7.3%	23.4%	0.00	0.20	0.18	-0.11	0.17	0.33	0.64	0.20	0.51	0.72	0.69	0.80	1							
(13) Private Equity	7.8%	22.7%	0.00	0.16	0.06	-0.17	0.11	0.29	0.61	0.17	0.62	0.76	0.73	0.70	0.64	1						
(14) REITs	4.9%	19.9%	0.00	0.30	0.28	0.07	0.27	0.39	0.64	0.29	0.49	0.71	0.69	0.65	0.56	0.58	1					
(15) Private Real Estate	3.8%	13.7%	0.00	0.18	0.15	-0.02	0.10	0.18	0.39	0.11	0.44	0.45	0.44	0.39	0.34	0.49	0.62	1				
(16) Hedge Funds	3.0%	7.9%	0.00	0.21	0.14	-0.17	0.16	0.36	0.65	0.19	0.57	0.74	0.74	0.73	0.68	0.63	0.56	0.39	1			
(17) Commodities	2.6%	18.0%	0.00	0.18	-0.05	-0.21	-0.04	0.11	0.36	0.06	0.34	0.33	0.34	0.41	0.41	0.33	0.28	0.21	0.43	1		
(18) Venture Capital	11.9%	29.5%	0.00	0.04	-0.10	-0.26	-0.03	0.19	0.59	-0.07	0.62	0.72	0.71	0.67	0.59	0.75	0.46	0.47	0.56	0.31	1	
(19) Infrastructure	5.0%	16.6%	0.00	0.29	0.20	-0.07	0.22	0.33	0.63	0.26	0.59	0.70	0.67	0.70	0.63	0.66	0.65	0.51	0.62	0.40	0.55	1

Table 4

Consistency Between Subjective Market Risk Premia and the Pricing of Subjective βs (Estimated Separately for each Combination of Year/Institution)

This table reports a test of the CAPM (Equation 5) under the subjective beliefs of our institutions. Specifically, we report the (cross-institution distribution of the) $\lambda_{j,t}^{(m)} - \mu_{j,t}^{(m)}$ values with their respective t-statistics for each year (in parentheses), which reflect tests for whether the price of risk each institution assigns to $\beta_{j,n,t}^{(m)}$ (i.e., $\lambda_{j,t}^{(m)}$) is consistent with the institution's subjective market expected return (i.e., $\mu_{j,t}^{(m)}$). We consider two models to determine βs. The first is the Equity CAPM ($m = e$), with market portfolio based on *US Equities (Large Cap)*. The second is the Pension CAPM ($m = p$), with market portfolio based on the aggregate allocation of US Public Pension Funds. Section 2.2 provides more details about our subjective beliefs data and Section 3.1 provides more details about the analysis reported in this table.

	Equity CAPM $\lambda_{j,t}^{(e)} - \mu_{j,t}^{(e)}$			Pension CAPM $\lambda_{j,t}^{(p)} - \mu_{j,t}^{(p)}$		
Year	**Lowest**	**Median**	**Highest**	**Lowest**	**Median**	**Highest**
1998	-1.6%	-0.1%	0.4%	-1.5%	-0.6%	0.3%
	(-1.57)	(0.07)	(0.37)	(-2.03)	(-0.36)	(0.55)
2000	-0.4%	-0.1%	1.3%	-0.7%	-0.4%	0.3%
	(-0.11)	(-0.09)	(1.56)	(-1.43)	(-0.29)	(0.15)
2002	-3.5%	-0.4%	1.0%	-2.7%	-0.7%	-0.1%
	(-1.62)	(-0.26)	(1.15)	(-2.00)	(-0.73)	(-0.12)
2004	-0.7%	0.6%	1.2%	-1.1%	-0.4%	0.0%
	(-1.27)	(0.43)	(1.31)	(-1.13)	(-0.81)	(0.09)
2006	-1.7%	-0.4%	1.3%	-1.9%	-1.0%	0.4%
	(-1.78)	(-0.56)	(1.51)	(-2.60)	(-1.81)	(0.77)
2008	-1.9%	-0.7%	-0.4%	-2.5%	-1.3%	-0.5%
	(-11.30)	(-0.66)	(-0.59)	(-3.44)	(-2.21)	(-0.98)
2010	-1.4%	-0.4%	0.5%	-1.5%	-0.9%	-0.4%
	(-2.14)	(-0.42)	(0.38)	(-5.41)	(-1.05)	(-0.59)
2012	-1.5%	0.1%	1.4%	-1.5%	-0.5%	0.3%
	(-2.96)	(0.15)	(0.70)	(-2.65)	(-0.99)	(0.91)
2014	-1.2%	0.1%	1.5%	-1.6%	-0.7%	0.5%
	(-2.13)	(0.13)	(1.34)	(-3.75)	(-1.21)	(0.85)
2016	-0.6%	0.2%	2.6%	-1.4%	-0.4%	1.8%
	(-2.44)	(0.29)	(1.84)	(-2.67)	(-0.81)	(1.55)
2018	-1.4%	0.5%	4.8%	-1.2%	-0.3%	2.3%
	(-1.88)	(0.73)	(1.22)	(-2.45)	(-0.37)	(0.97)
2020	-4.5%	0.4%	2.8%	-4.0%	-0.5%	1.9%
	(-1.15)	(0.46)	(1.19)	(-1.49)	(-0.84)	(1.30)
2022	-1.2%	0.1%	2.2%	-1.3%	-0.8%	1.3%
	(-1.44)	(0.07)	(1.16)	(-2.89)	(-1.41)	(1.64)
Average	-1.7%	0.0%	1.8%	-1.9%	-0.7%	0.9%
	(-2.08)	(0.06)	(1.03)	(-2.82)	(-1.17)	(0.78)

Table 5

Fraction of Expected Return Variation Originating from Risk Premia vs Alphas (Estimated Jointly using all Years, Institutions, and Asset Classes)

This table reports the fraction of subjective expected returns ($\mu_{j,n,t} = \mathbb{E}_{j,t}[r_n]$) variability explained by subjective risk premia ($rp^{(m)}_{j,n,t} = \beta^{(m)}_{j,n,t} \cdot \mu^{(m)}_{j,t}$) and subjective pricing errors ($\alpha^{(m)}_{j,n,t}$) based on the decomposition in Equation 7. Different columns add different fixed effects to the regressions underlying this decomposition. We consider two models to determine βs. The first is the Equity CAPM ($m = e$), with market portfolio based on *US Equities (Large Cap)*. The second is the Pension CAPM ($m = p$), with market portfolio based on the aggregate allocation of US Public Pension Funds. Section 2.2 provides more details about our subjective beliefs data and Section 3.1 provides more details about the analysis reported in this table.

Identification from Variation Across =	**Multiple Sources**				**Asset Classes**		**Institutions**		**Years**	
	[1]	**[2]**	**[3]**	**[4]**	**[5]**	**[6]**	**[7]**	**[8]**	**[9]**	**[10]**
Equity CAPM										
% of μ Variation from Risk Premia	76%	76%	76%	52%	77%	77%	50%	47%	47%	49%
% of μ Variation from Alphas	24%	24%	24%	48%	23%	23%	50%	53%	53%	51%
Pension CAPM										
% of μ Variation from Risk Premia	91%	91%	91%	63%	92%	92%	62%	56%	56%	61%
% of μ Variation from Alphas	9%	9%	9%	37%	8%	8%	38%	44%	44%	39%
Year Fixed Effect		X			X		X			
Institution Fixed Effect			X		X				X	
Asset Class Fixed Effect				X			X		X	
Year×Institution Fixed Effect						X				
Year×Asset Class Fixed Effect								X		
Institution×Asset Class Fixed Effect										X

Table 6
Realized Return Data: Indice Names, Data Sources, and Sample Period

This table provides a list of the indices used for the realized returns matched to each asset class present in our beliefs data (i.e., the CMAs of our institutions).

CMA Asset Class	Index Name	Data Source	First Return Date	Last Return Date
US Cash	3-Month Treasury Bill Return	CRSP	04/1957	12/2022
US TIPS	Bloomberg Barclays US Treasury Inflation Notes	Bloomberg	03/1997	12/2022
US Bonds	Bloomberg Barclays US Aggregate Bond Index	Bloomberg	01/1976	12/2022
US Govt Bonds	Bloomberg Barclays US Treasury Index	Bloomberg	01/1973	12/2022
US Muni Bonds	ICE Bank of America US Municipal Securities Index	Bloomberg	01/1980	12/2022
US Inv Grade Corp Bonds	Bloomberg Barclays US Investment Grade Index	Bloomberg	01/1973	12/2022
US High Yield Corp Bonds	Bloomberg Barclays US High Yield Index	Bloomberg	07/1983	12/2022
Global Bonds (Ex US)	Bloomberg Barclays Global Aggregate- Ex US	Bloomberg	01/1990	12/2022
Private Debt	-	-	-	-
US Equities (Large Cap)	S&P 500 Index	CRSP & Bloomberg[a]	04/1957	12/2022
US Equities (Small Cap)	Russell 2000 Index	Bloomberg	01/1979	12/2022
Global Equities (Dev, Ex US)	MSCI World ex US Index	Bloomberg	02/1970	12/2022
Global Equities (Emerging)	MSCI Emerging Markets	Bloomberg	01/1988	12/2022
Private Equity	-	-	-	-
REITs	FTSE NAREIT US Real Estate Index (All Equity)	NAREIT	01/1972	12/2022
Private Real Estate	NCREIF Value Weighted Index	NCREIF	Q1/1978	Q4/2022
Hedge Funds	HFRI FOF: Diversified Index	Hedge Fund Research	01/1990	12/2022
Commodities	S&P GSCI or Bloomberg Commodity Index	Bloomberg	02/1970	12/2022
Venture Capital	-	-	-	-
Infrastructure	Dow Jones Brookfield Global Infrastructure Index	Bloomberg	01/2003	12/2022

[a]The S&P 500 inception date is March 4, 1957 (so, the first complete monthly return is 04/1957). However, Bloomberg only has total return data for the S&P500 starting in 02/1970. So, we use CRSP for the S&P 500 monthly returns from 04/1957 to 01/1970. CRSP only reports ex-dividend returns for the S&P 500 ($R^{ex}_{SP,t}$). To obtain the S&P 500 total return from CRSP, we use $R_{SP,t} = R^{(ex)}_{SP,t} + (R_{vw,t} - R^{(ex)}_{vw,t})$, where $R_{vw,t}$ and $R^{(ex)}_{vw,t}$ are the cum- and ex-dividend returns for the CRSP value-weighted index. Over the period we directly observe $R_{SP,t}$ from Bloomberg, the correlation between the two $R_{SP,t}$ values (from CRSP and Bloomberg) is 99.95%. Moreover, the difference in average annualized returns is only 0.04%. As such, the $R_{SP,t}$ imputed from CRSP data seems to be a reliable proxy for S&P 500 returns. Note that series commonly used in the literature for the period before 1957 (e.g., Robert Shiller's S&P 500 returns) do not actually refer to the S&P 500 before March 4, 1957 (the inception date).

Table 7
Predicting Realized Returns, Risk, and Alphas using Subjective Beliefs

This table reports results from the estimation of equations analogous to Equation 16. $\mu_{n,t}$, $\sigma^2_{n,t}$, $\beta^{(e)}_{n,t}$, and $\alpha^{(e)}_{n,t}$ are the belief quantities aggregated across institutions as described in Subsection 4.1. $\mu^{(hy)}_{n,t}$ and $\alpha^{(e,hy)}_{n,t}$ are linear transformations of $\mu_{n,t}$ and $\alpha^{(e)}_{n,t}$ to reflect h-year expected average returns and alphas, respectively (as per Equation 18 and Footnote 25). We consider pooled regressions (Variation=All), regressions with year fixed effects (Variation=Across AC), and regressions with asset class fixed effects (Variation=Over Time). Inference relies on double clustering (by year and asset class). $p_{a=0,b=1}$ reflects the p-value for a Wald test of the joint hypothesis that $a = 0$ and $b = 1$. R^2 reflects the pooled R^2 in the Variation=All case and within R^2 values in the other two cases. $R^2_{a=0,b=1}$ reflects an R^2 metric design to identify whether belief variation (across asset classes and over time) improves upon belief homogeneity (see Footnote 22). R^2_{OOS} reflects an out-of-sample R^2 metric, which evaluates the predictability of the given predictor against the predictability of a historical returns benchmark (see Footnote 23). The realized risk and alphas metrics in Panel B are calculated from monthly returns within year $t + 1$ (with alphas multiplied by 12). Sections 2.2 and 4.1 provide more details about the data and Section 4.3 provides more details about the analysis reported in this table.

PANEL A - Predicting Realized Returns

	$r_{n,t+1} = a + b \cdot \mu_{n,t} + \epsilon_t$			$r_{n,t+1} = a + b \cdot \mu^{(1y)}_{n,t} + \epsilon_t$			$\bar{r}_{n,t\to t+3} = a + b \cdot \mu^{(3y)}_{n,t} + \epsilon_t$		
Variation =	**All**	**Across AC**	**Over Time**	**All**	**Across AC**	**Over Time**	**All**	**Across AC**	**Over Time**
a	0.00			0.00			0.01		
$(t_{a=0})$	(-0.37)			(0.15)			(1.40)		
b	1.22	1.12	4.83	1.04	0.82	1.11	0.97	0.94	0.82
$(t_{b=0})$	(2.69)	(2.04)	(1.90)	(2.82)	(2.38)	(1.92)	(5.35)	(5.96)	(1.50)
$[t_{b=1}]$	[0.49]	[0.21]	[1.51]	[0.10]	[-0.54]	[0.20]	[-0.18]	[-0.40]	[-0.34]
$\{p_{a=0,b=1}\}$	{0.87}			{0.96}			{0.33}		
R^2	3.8%	4.9%	4.6%	7.1%	5.5%	5.3%	12.7%	14.2%	3.9%
$R^2_{a=0,b=1}$	3.6%	4.0%	1.7%	7.0%	7.4%	5.2%	11.8%	12.3%	2.5%
R^2_{OOS}		5.5%	5.8%		12.6%	12.9%		13.6%	10.9%

PANEL B - Predicting Realized Risk and Alphas

	$\widehat{\sigma}^2_{n,t+1} = a + b \cdot \sigma^2_{n,t} + \epsilon_t$			$\widehat{\beta}^{(e)}_{n,t+1} = a + b \cdot \beta^{(e)}_{n,t} + \epsilon_t$			$\widehat{\alpha}^{(e)}_{n,t+1} = a + b \cdot \alpha^{(e,1y)}_{n,t} + \epsilon_t$		
Variation =	**All**	**Across AC**	**Over Time**	**All**	**Across AC**	**Over Time**	**All**	**Across AC**	**Over Time**
a	0.00			0.02			0.00		
$(t_{a=0})$	(0.17)			(0.51)			(0.40)		
b	0.68	0.71	-0.28	0.96	0.99	-0.24	0.32	0.31	0.38
$(t_{b=0})$	(9.45)	(6.95)	(-0.54)	(16.82)	(16.86)	(-1.21)	(0.61)	(0.61)	(0.66)
$[t_{b=1}]$	[-4.51]	[-2.81]	[-2.44]	[-0.72]	[-0.20]	[-6.15]	[-1.28]	[-1.40]	[-1.09]
$\{p_{a=0,b=1}\}$	{0.00}			{0.76}			{0.44}		
R^2	26.8%	38.8%	0.6%	61.5%	67.9%	1.3%	0.5%	0.5%	0.5%
$R^2_{a=0,b=1}$	19.3%	18.6%	-9.3%	61.4%	61.7%	-25.3%	-1.8%	-1.1%	-1.8%
R^2_{OOS}		9.5%	-26.1%		60.9%	-14.8%		5.4%	5.8%

Table 8

Predicting Realized Returns using Subjective $\mathbb{E}[\boldsymbol{Return}]$ (by Asset Class)

This table reports results from the estimation of Equation 16 by asset class. $\mu_{n,t}$ is the subjective expected return aggregated across institutions as described in Subsection 4.1. The predictor in these regressions, $\mu_{n,t}^{(1y)}$, is a linear transformations of $\mu_{n,t}$ to reflect 1-year expected returns (as per Equation 18). Inference relies on Newey and West (1987, 1994). R^2_{OOS} reflects an out-of-sample R^2 metric, which evaluates the predictability of the given predictor against the predictability of the historical average return for the given asset class (see Footnote 23). N_{R^2} reflects the number of years of forecasting errors we have to compute R^2_{OOS}. Sections 2.2 and 4.1 provide more details about the data and Section 4.3 provides more details about the analysis reported in this table.

	a	$(t_{a=0})$	b	$(t_{b=0})$	$[t_{b=1}]$	R^2	R^2_{OOS}	N_{R^2}
US TIPS	0.03	(1.61)	0.34	(0.60)	[-1.18]	2.9%	9.7%	5
US Bonds	0.01	(0.33)	1.01	(1.48)	[0.02]	4.9%	4.2%	27
US Govt Bonds	0.02	(1.30)	0.55	(1.42)	[-1.16]	10.0%	1.8%	24
US Muni Bonds	0.01	(1.44)	1.09	(4.06)	[0.34]	14.7%	10.2%	19
US Inv Grade Corp Bonds	0.02	(0.91)	1.33	(3.83)	[0.96]	46.6%	39.6%	17
US High Yield Corp Bonds	-0.04	(-1.52)	2.51	(4.84)	[2.91]	59.3%	45.3%	19
Global Bonds (Ex US)	0.00	(0.08)	0.68	(0.66)	[-0.32]	1.3%	15.7%	13
US Equities (Large Cap)	0.03	(0.30)	0.98	(0.67)	[-0.02]	1.7%	3.1%	35
US Equities (Small Cap)	0.05	(0.59)	0.27	(0.24)	[-0.66]	0.2%	11.6%	24
Global Equities (Developed, Ex US)	-0.09	(-0.85)	2.32	(1.67)	[0.95]	8.6%	8.2%	25
Global Equities (Emerging)	0.14	(0.88)	-0.62	(-0.37)	[-0.96]	0.9%	15.9%	15
REITs	-0.05	(-0.61)	2.71	(2.32)	[1.46]	16.8%	10.2%	25
Private Real Estate	-0.01	(-0.39)	1.08	(2.99)	[0.21]	15.3%	21.4%	25
Hedge Funds	-0.01	(-0.15)	0.86	(0.87)	[-0.14]	3.3%	-5.7%	13
Commodities	-0.10	(-1.97)	2.15	(1.24)	[0.66]	11.3%	4.7%	19
Infrastructure	-0.02	(-0.34)	1.43	(1.94)	[0.58]	15.6%	-	-

Internet Appendix

"The Subjective Risk and Return Expectations of Institutional Investors"

By Spencer J. Couts, Andrei S. Gonçalves, and Johnathan A. Loudis

This Internet Appendix is organized as follows. Section A provides a full description of the data sources and measurement for the analysis and Section B describes additional results that supplement the main findings in the paper.

A Data Details

This section provides details on our collection of Capital Market Assumptions (CMAs).

A.1 Data Collection Process

We collected the long-term CMAs of 34 institutions on total: 18 asset managers and 16 institutional investor consultants. We rely on three complementary data collection approaches:

(i) The bulk of the data comes directly from the institutions covered in our CMAs. We identify individuals within each institution that are connected to the production of CMAs and contact them directly to request access to their data. In all successful cases, the institution send us continuous (at least annual) data covering from some initial year until their most recent CMA, and thus these data are free of issues related to attrition rates.

(ii) To complement the data sent directly by institutions, we also obtain data from online sources. As in Dahlquist and Ibert (2023), our approach is simple: we obtain the most recent CMA of each institution directly from their website and complement these CMAs with prior CMAs obtained through google searches and `archive.org`.

(iii) The third data collection approach is based on indirect data obtained from pension funds. Specifically, we contacted various pension funds to request the CMAs they rely on when deciding their portfolio allocation. In the successful cases, the pension funds send us their their portfolio allocation reports and/or their CMA reports. In turn, these reports include the numbers (expected returns, volatilities, and correlations) associated with the CMAs of third party institutions that the pension fund rely on (including consultants and sometimes asset managers).

Whenever an institution-year observation is available through more than one of the three methods above, we rely on a pecking order for data selection (we use (i) when available, (ii) when (i) is not available, and (iii) when nether (i) nor (ii) is available). Given this pecking order, 82% of the institution-year observations in our baseline sample is based on data collected from methods (i)+(ii).

As we explain in Section B, while the results we report in the main text combine consultants with managers and rely on this pecking order, we explore different subsets of the data in our robustness checks to demonstrate that our results are not due to particular biases that may be present in the data. For example, we show that our results are similar if we rely only on consultants or only on managers, indicating that our results are not due to any particular incentive that is specific to either of these type of institutions. Analogously, we show that our results are similar if we rely only on data collection process (i)+(ii) or only on data collection process (iii), indicating that our results are not due to potential biases that may arise from collecting data directly from the institution or collecting data indirectly through pension funds.

Each institution-year CMA covers a range of asset classes, which we identify based on the asset class name used in the CMA report and/or the actual portfolio index stated in the CMA report. Our final sample contains a risk-free asset class proxy (*US Cash*) as well as 19 risky asset classes. To decide on these 19 risky asset classes, we consider three aspects. First, whether the asset class is a major asset class for institutional investors. Second, whether the asset class is covered by a reasonable number of institutions in our sample. And third, whether the asset class is covered over a reasonable time period within the institutions that cover it. We include all asset classes that perform well in these dimensions, with the final list of asset classes covered in our study available in the first column of Table IA.1.

Since the names/indexes for different asset classes differ across institutions and over time within institutions, we use our judgment when mapping asset classes within and across institutions to the asset classes included in our final sample. Specifically, we start by manually mapping each asset class in each institution-year observation to an institution-specific asset class name (fixed over time) that reflects the underlying asset class well. Then, we map each institution-specific asset class name to a broader asset class name (which we refer to as the master asset class) that reflects the institution-specific asset class name reasonably well while allowing for small mismatches to accommodate asset classes from different institution under the same asset class name. Finally, we map these master asset classes to our final 20 asset classes in column 1 of Table IA.1 (which we refer to as primary asset classes). Note that any mismatch between asset classes across institutions would lead to an overstatement of heterogeneity across institutions, but one of our main findings is that belief heterogeneity across asset classes dominates belief heterogeneity across institutions.

The default master asset class of each primary asset class is the one with the exact same name as the primary asset class itself. Column "% Match" in Table IA.1 shows the fraction of institution-year observations with the primary asset class available that has the name of the primary asset class matching the name of the underlying master asset class. These numbers are high but not 100% because we allow for substitutions of the master asset class when the default one is not available. The other columns of Table IA.1 show the master asset classes that are used as substitutes for the default master asset class. We allow for some flexibility (to increase coverage) without allowing for too large of a deviation between the primary asset class and its underlying master asset class. Section B.1.3 provides a robustness analysis that focuses on institution-year observations for which the primary asset class name matches the name of the underlying master asset class used (the results are very similar to the ones reported in the main text).

A.2 Extracting Beliefs from the CMAs

As JP Morgan details in their 2015 report, their expected returns (μ), expected volatilities (σ), and expected correlations (ρ) are all obtained based on their views on log returns through a log-Normal transformation.[IA.1] Specifically, they first form their beliefs in log return space and then translate to raw return space using the assumption that log returns are normally distributed. In mathematical terms, they report (for each asset class j)

[IA.1] The use of μ in this section embeds a slight abuse of notation since μ reflects expected excess returns in the main text while it reflects expected returns (not in excess of the risk-free asset) in this section.

$$\begin{pmatrix} \mu_j \\ \sigma_j^2 \\ \rho_{j,i} \end{pmatrix} = \begin{pmatrix} e^{\widehat{\mu}_j + \frac{1}{2}\cdot\widehat{\sigma}_j^2} - 1 \\ e^{2\cdot(\widehat{\mu}_j + \frac{1}{2}\cdot\widehat{\sigma}_j^2)} \cdot (e^{\widehat{\sigma}_j^2} - 1) \\ \left(e^{\widehat{\rho}_{j,i}\cdot\widehat{\sigma}_i\cdot\widehat{\sigma}_j} - 1\right) / \sqrt{(e^{\widehat{\sigma}_j^2} - 1)\cdot(e^{\widehat{\sigma}_i^2} - 1)} \end{pmatrix} \tag{IA.1}$$

where $\widehat{\mu}$, $\widehat{\sigma}$, and $\widehat{\rho}$ are the expected log returns, expected log volatilities, and expected log correlations.[IA.2]

To obtain the log return estimates, one needs to solve the system above for the log return quantities, which yields:

$$\begin{pmatrix} \widehat{\sigma}_j^2 \\ \widehat{\mu}_j \\ \widehat{\rho}_{j,i} \end{pmatrix} = \begin{pmatrix} log\left(1 + \sigma_j^2/(\mu_j + 1)^2\right) \\ log(\mu_j + 1) - \frac{1}{2}\cdot\widehat{\sigma}_j^2 \\ log\left(1 + \rho_{j,i}\cdot\sqrt{(e^{\widehat{\sigma}_j^2} - 1)\cdot(e^{\widehat{\sigma}_i^2} - 1)}\right) / (\widehat{\sigma}_i\cdot\widehat{\sigma}_j) \end{pmatrix} \tag{IA.2}$$

All institution-year observations in our sample contain σ_j and $\rho_{j,i}$. However, they vary on whether they contain "expected arithmetic returns" (which is μ in our notation), "expected geometric returns" (which is $e^{\widehat{\mu}_j} - 1$ in our notation as per Footnote IA.2), or both. Using the transformations above, we recover all six variables (μ_j, σ_j, $\rho_{j,i}$, $\widehat{\mu}_j$, $\widehat{\sigma}_j$, $\widehat{\rho}_{j,i}$) for all institution-year observations in our sample and use subsets of these variables for different parts of our analysis. In particular, the main text is entirely based on (μ_j, σ_j, $\rho_{j,i}$) while our robustness check related to "expected geometric returns" (in Section B.2.1) also uses $\widehat{\mu}_j$ and $\widehat{\sigma}_j$. Below, we describe how we recover all six variables for the different cases that arise in our data construction. We then describe which variables are used in the main text and in our robustness checks.

Case 1: We have "Expected Arithmetic Return"

In this case, we directly observe (μ_j, σ_j, $\rho_{j,i}$) and solve for ($\widehat{\mu}_j$, $\widehat{\sigma}_j$, $\widehat{\rho}_{j,i}$) from the system in Equation IA.2.

[IA.2] The JP Morgan 2015 report also provides $e^{\widehat{\mu}_j} - 1$ as the "Expected Compound Return".

Case 2: We have "Expected Geometric Return"

In this case, we directly observe $(\sigma_j,\ \rho_{j,i})$ and directly recover $\widehat{\mu}_j = log(1 +$ Expected Geometric Return). We then combine Equations IA.1 and IA.2 to solve for $(\mu_j,$ $\widehat{\sigma}_j,\ \widehat{\rho}_{j,i})$ from

$$\begin{pmatrix} \mu_j \\ \widehat{\sigma}_j^2 \\ \widehat{\rho}_{j,i} \end{pmatrix} = \begin{pmatrix} e^{\widehat{\mu}_j + \frac{1}{2}\cdot\widehat{\sigma}_j^2} - 1 \\ log\left(1 + \sigma_j^2/(\mu_j+1)^2\right) \\ log\left(1 + \rho_{j,i}\cdot\sqrt{(e^{\widehat{\sigma}_j^2}-1)\cdot(e^{\widehat{\sigma}_i^2}-1)}\right) / \left(\widehat{\sigma}_i\cdot\widehat{\sigma}_j\right) \end{pmatrix} \tag{IA.3}$$

The first two equations in this system cannot be solved in closed form for μ_j and $\widehat{\sigma}_j$ as functions of $\widehat{\mu}_j$ and σ_j. So, we solve these two equations numerically using a root-solving algorithm and obtain $\widehat{\rho}_{j,i}$ from the third equation in closed-form.

Case 3: We have "Expected Arithmetic Return" & "Expected Geometric Return"

In this case, we directly observe $(\mu_j,\ \sigma_j,\ \rho_{j,i})$ and directly recover $\widehat{\mu}_j = log(1 +$ Expected Geometric Return). We then calculate $(\widehat{\sigma}_j,\ \widehat{\rho}_{j,i})$ from

$$\begin{pmatrix} \widehat{\sigma}_j^2 \\ \widehat{\rho}_{j,i} \end{pmatrix} = \begin{pmatrix} log\left(1 + \sigma_j^2/(e^{2\cdot\widehat{\mu}_j + \widehat{\sigma}_j^2})\right) \\ log\left(1 + \rho_{j,i}\cdot\sqrt{(e^{\widehat{\sigma}_j^2}-1)\cdot(e^{\widehat{\sigma}_i^2}-1)}\right) / \left(\widehat{\sigma}_i\cdot\widehat{\sigma}_j\right) \end{pmatrix} \tag{IA.4}$$

The first equation in this system cannot be solved in closed form for $\widehat{\sigma}_j$ as a function of $\widehat{\mu}_j$ and σ_j. So, we solve this equation numerically using a root-solving algorithm and obtain $\widehat{\rho}_{j,i}$ from the second equation in closed-form. Note that this approach ensures the "Expected Arithmetic Return" is not used when calculating $(\widehat{\sigma}_j,\ \widehat{\rho}_{j,i})$.

Our use of $(\mu_j,\ \sigma_j,\ \rho_{j,i},\ \widehat{\mu}_j,\ \widehat{\sigma}_j,\ \widehat{\rho}_{j,i})$

In the main text, we construct $\sum_{j,t}^{R}$ from $(\sigma_{j,t},\ \rho_{j,i,t})$ and set $\mathbb{E}_{j,t}[R] = \mu_{j,t}$, which does not rely on the "Expected Geometric Return" variable (unless the "Expected Arithmetic Return" variable is missing). In our robustness check of Section B.2.1, we still construct $\sum_{j,t}^{R}$ from $(\sigma_{j,t},\ \rho_{j,i,t})$, but we set $\mathbb{E}_{j,t}[R] = e^{\widehat{\mu}_{j,t}} - 1$ based on Footnote IA.2, which does not rely on the

“Expected Arithmetic Return” variable (unless the “Expected Geometric Return” variable is missing). Section B.2.1 also explores $\mathbb{E}_{j,t}[R] = e^{\widehat{\mu}_{j,t}+0.5\cdot\widehat{\sigma}^2_{j,t}} - 1$, but the results are very similar, and thus omitted for brevity.

B Results from Alternative Specifications

This section provides results that complement the main findings in the paper.

B.1 Main Results with Alternative Subsets of the Data

In the main text, we combine different data sources and institution types to maximize data coverage (with the measurement of subjective beliefs detailed in Section A.1). In this section, we show that our results are similar for different subsets of the data. This finding helps to alleviate/eliminate concerns related to different potential biases that may affect our data.

B.1.1 Main Results for Each Institution Type

One potential worry with including institutional investor consultants in our analysis is that they may have distortionary incentives related to their business model. For instance, it is possible that the CMAs of consultants are distorted to justify the portfolio allocation of their clients. To address this issue, Figure IA.2 shows that our main results are very similar if we rely on a sample that is based entirely on asset managers.

Relatedly, a potential worry with including managers in our analysis is that they may distort their CMAs to meet their own incentives. For instance, managers may create CMAs that are design to create demand for the asset classes they currently overweight in their portfolios. To address this issue, Figure IA.3 shows that our main results are very similar if we rely on a sample that is based entirely on consultants.

The results in this subsection indicate that our results are not due to any particular incentive that is specific to either consultants or managers.

B.1.2 Main Results for Each Data Source

One may worry that pension fund reports do not reflect the true CMAs of the institutions we are trying to capture. This would happen, for example, if institutions sent different CMAs to different pension funds based on the pension funds' own incentives (e.g., high expected returns allow pension funds to discount their liabilities at a high rate, with this incentive depending on how underfunded the pension fund is). To address this issue, Figure IA.4 shows that our main results are very similar if we rely on a sample that is entirely based on the direct CMAs of the underlying institutions (data collection processes (i) and (ii) in Section A.1).

Relatedly, one may worry that institutions have incentives to send us distorted CMAs that make them "look good". For instance, they could send us CMAs that combine their current models with historical data, and thus that differ from the CMAs produced historically (which better reflect the real-time beliefs of the institutions). To address this issue, Figure IA.5 shows that our main results are very similar if we rely on a sample that is entirely based on the CMAs we receive from pension funds (data collection processes (iii) in Section A.1).

B.1.3 Main Results Using Observations in which the Primary Asset Class is Available

As Table IA.1 highlights, to increase coverage, we build our 20 primary asset classes by combining data from different master key asset classes depending on data availability. This process creates mismatches between the asset classes of different institutions. To address this issue, Figure IA.6 shows that our main results are very similar if we rely on a sample that focuses on institution-year observations for which the primary asset class name matches the name of the underlying master asset class used (that is, a sample that does not rely on the substitute asset classes in Table IA.1). The exception is that we still rely on substitute asset classes for *US Cash* and *Equities (Large Cap)* since these two asset classes are needed to obtain μ and β.

B.2 Main Results with Alternative Constructions of Subjective Beliefs

The results in the main text are based on institutions' beliefs about arithmetic (excess) returns and without any homogeneity restriction on the horizon of beliefs (i.e., different institutions report beliefs for different horizons in the baseline analysis). In this section, we show that our conclusions are very similar if we instead (i) use expected geometric returns to recover $\mathbb{E}[R]$ or (ii) focus only on institution-year observations with a horizon of 10 years, which is the most common horizon in our dataset (capturing around 41% of our institution-year observations).

B.2.1 $\mathbb{E}[R]$ Based on Expected Geometric Returns

In our baseline analysis, $\mathbb{E}_{j,t}[R]$ is based on expected arithmetic returns ($\mu_{j,t}$ in the notation of Section A.2). To consider expected geometric returns, we reproduce our main results using $\mathbb{E}_{j,t}[R] = e^{\widehat{\mu}_{j,t}} - 1$ (while holding all other empirical decisions fixed), where $\widehat{\mu}_{j,t}$ is the expected log return (see the discussion in Section A.2). Figure IA.7 provides the results, which demonstrate that our findings are very similar if we use expected geometric returns instead of expected arithmetic returns. We also consider arithmetic returns implied from the geometric returns (e.g., $\mathbb{E}_{j,t}[R] = e^{\widehat{\mu}_{j,t}+0.5\cdot\widehat{\sigma}^2_{j,t}} - 1$), but the results are very similar, and thus omitted for brevity.

B.2.2 Beliefs Based on a Homogeneous Horizon

In our baseline analysis, the beliefs have heterogeneous horizons (they vary from 4 years to 30 years). To ensure our results are not coming from horizon heterogeneity, we consider a sample in which all institution-year observations have a 10 year horizon, which is the most common horizon in our dataset (capturing around 41% of our institution-year observations). Figure IA.7 provides the results, which demonstrate that our findings are very similar if we rely on a sample in which belief horizons are homogeneous.

B.3 Main Results with Aggregated Data Obtained from a Third Party Company

While our robustness checks of Sections B.1 and B.2 are useful in minimizing several concerns that readers may have about our dataset, it is always possible that our dataset is subject to other biases that we have not addressed. This section explores a different dataset (of aggregated CMAs) obtained from a third party company.

This third party company systematically collects (annually) the CMAs of many institutional investor consultants and asset managers. We have access to their annual CMA surveys from 2012 to 2022, with each year providing aggregated (across institutions) data on expected returns, volatilities, and correlations. Their CMA surveys cover 17 institutions in 2012, with this number growing over time to 40 institutions in 2022.

Since we do not observe the institution-level data in this case, we treat each aggregate observation (of expected returns, volatiltities, and correlations) as coming from a single institution. Then, we perform our main analysis using this single representative institution (except that we cannot produce results related to heterogeneity across institutions). Figure IA.9 provides the main results. In a nutshell, the overall findings are very similar to what we report in the main text.

References for Internet Appendix

Dahlquist, M. and M. Ibert (2023). "Equity Return Expectations and Portfolios: Evidence from Large Asset Managers". Working Paper.

Table IA.1
Constructing our Asset Classes

This table reports the master asset classes we rely on when constructing the primary asset classes used in our paper. Specifically, we start by manually mapping each asset class in each institution-year observation to an institution-specific asset class name (fixed over time) that reflects the underlying asset class well. Then, we map each institution-specific asset class name to a broader asset class name (which we refer to as the master asset class) that reflects the institution-specific asset class name reasonably well while allowing for small mismatches to accommodate asset classes from different institution under the same asset class name. Finally, we map these master asset classes to our final 20 asset classes (which we refer to as primary asset classes). The default master asset class of each primary asset class is the one with the exact same name as the primary asset class itself (in the first column of this table). Column "% Match" shows the fraction of institution-year observations with the primary asset class available that has the name of the primary asset class matching the name of the underlying master asset class. The other columns show the master asset classes that are used as substitutes for the default master asset class. Sections 2 and A provide more details about our subjective beliefs data.

		Substitute Asset Classes			
Major Asset Class	**% Match**	**Substitute 1**	**Substitute 2**	**Substitute 3**	**Substitute 4**
US Cash	92%	3-Month Libor	US Treasuries ST		
US TIPS	100%				
US Bonds	86%	US Bonds Core+			
US Govt Bonds	58%	US Treasuries MT	US Govt MT	US Treasuries LT	US Govt LT
US Muni Bonds	80%	US Muni Bonds (1-15 Years)			
US Inv Grade Corp Bonds	69%	US Corp Fixed Income	US Corp Bonds LT		
US High Yield Corp Bonds	100%				
Global Bonds (Ex US)	50%	Govt Bonds (Ex US)	Inv Grade Corp Bonds (Ex US)	Global Bonds	Global Govt Bonds
Private Debt	100%				
US Equities (Large Cap)	70%	US Equities			
US Equities (Small Cap)	81%	US Equities (Mid+Small Cap)			
Global Equities (Dev, Ex US)	61%	Global Equities (Ex US)	Global Equities		
Global Equities (Emerging)	100%				
Private Equity	100%				
REITs	91%	REITs (Global)			
Private Real Estate	87%	Private Real Estate Funds	Real Estate (Global)		
Hedge Funds*	94%	Equal-Weighted Average of Different Hedge Fund Strategies			
Commodities	97%	Commodity Futures			
Venture Capital	100%				
Infrastructure	73%	Private Infrastructure			

Abbreviations: ST = Short-Term; MT = Mid-Term; LT = Long-Term; Ex = Excluding; Cap = Market Capitalization; Govt = Government; Muni = Municipal;

* For Hedge Funds, we take the average of all available hedge fund strategies when the major category (Hedge Funds) is missing.

The hedge fund strategies are (they vary by institution-year observation) "Funds of Funds", "Multi Strategy", "Discretionary", "Open Mandate", "Directional", "Non-Directional", "Event Driven", "Market Neutral", "Relative Value", "Long-Short", "Long Bias", "Macro", "CTA", "Equity Style", "Credit Style Bonds Hedged", "Asymmetric Style", "Equity Hedged", "Convertible", "Moderate Aggregate Risk", "Absolute Returns", "Managed Futures"

Table IA.2
Predicting Realized Returns and Risk using Subjective Beliefs (Alternative Specifications)

This table reports results from the estimation of equations analogous to Equation 16. $\mu_{n,t}$ and $\beta^{(e)}_{n,t}$ are the belief quantities aggregated across institutions, with the aggregation method for Panel A as described in Subsection 4.1 and for Panel B as the average of the respective belief quantity across all institutions in a given year. $\mu^{(1y)}_{n,t}$ is a linear transformations of $\mu_{n,t}$ to reflect 1-year expected returns (as per Equation 18). We consider pooled regressions (Variation=All), regressions with year fixed effects (Variation=Across AC), and regressions with asset class fixed effects (Variation=Over Time). Inference relies on double clustering (by year and asset class). $p_{a=0,b=1}$ reflects the p-value for a Wald test of the joint hypothesis that $a = 0$ and $b = 1$. R^2 reflects the pooled R^2 in the Variation=All case and within R^2 values in the other two cases. $R^2_{a=0,b=1}$ reflects an R^2 metric design to identify whether belief variation (across asset classes and over time) improves upon belief homogeneity (see Footnote 22). R^2_{OOS} reflects an out-of-sample R^2 metric, which evaluates the predictability of the given predictor against the predictability of a historical returns benchmark (see Footnote 23). $\widehat{\beta}^{(e)}_{n,t}$ values are calculated from monthly returns within year $t + 1$. In Panel A, year $t + 1$ actually refers to the 12 months covering from the second quarter of year $t + 1$ to the first quarter of year $t + 2$. Sections 2.2 and 4.1 provide more details about the data and Section 4.3 provides more details about the analysis reported in this table.

PANEL A - Realized Returns from Q2 of $t + 1$ to Q1 of $t + 2$

	$r_{n,t+1} = a + b \cdot \mu_{n,t} + \epsilon_t$			$r_{n,t+1} = a + b \cdot \mu^{(1y)}_{n,t} + \epsilon_t$			$\widehat{\beta}^{(e)}_{n,t+1} = a + b \cdot \widehat{\beta}^{(e)}_{n,t} + \epsilon_t$		
Variation =	**All**	**Across AC**	**Over Time**	**All**	**Across AC**	**Over Time**	**All**	**Across AC**	**Over Time**
a	0.00			0.01			-0.01		
$(t_{a=0})$	(-0.25)			(0.30)			(-0.22)		
b	1.45	1.33	5.42	1.17	0.91	1.20	0.98	1.01	-0.22
$(t_{b=0})$	(2.12)	(1.70)	(1.57)	(2.21)	(2.10)	(1.55)	(18.96)	(18.65)	(-1.00)
$[t_{b=1}]$	[0.66]	[0.42]	[1.28]	[0.32]	[-0.20]	[0.26]	[-0.31]	[0.16]	[-5.61]
$\{p_{a=0,b=1}\}$	{0.79}			{0.78}			{0.85}		
R^2	3.7%	4.9%	4.1%	6.2%	4.8%	4.3%	61.4%	66.4%	0.9%
$R^2_{a=0,b=1}$	3.3%	3.6%	1.3%	6.0%	6.3%	4.1%	61.3%	61.7%	-22.7%
R^2_{OOS}		4.6%	4.2%		9.7%	9.2%		58.5%	-15.7%

PANEL B - Belief Aggregation Based on Annual Cross-Sectional Averages

	$r_{n,t+1} = a + b \cdot \mu_{n,t} + \epsilon_t$			$r_{n,t+1} = a + b \cdot \mu^{(1y)}_{n,t} + \epsilon_t$			$\widehat{\beta}^{(e)}_{n,t+1} = a + b \cdot \widehat{\beta}^{(e)}_{n,t} + \epsilon_t$		
Variation =	**All**	**Across AC**	**Over Time**	**All**	**Across AC**	**Over Time**	**All**	**Across AC**	**Over Time**
a	0.00			0.01			0.02		
$(t_{a=0})$	(-0.14)			0.45			0.88		
b	1.15	1.06	3.57	0.86	0.61	0.84	0.97	1.01	-0.22
$(t_{b=0})$	(2.50)	(1.92)	(1.48)	(2.09)	(1.61)	(1.27)	(17.43)	(18.26)	(-0.95)
$[t_{b=1}]$	[0.33]	[0.11]	[1.06]	[-0.34]	[-1.03]	[-0.25]	[-0.56]	[0.17]	[-5.29]
$\{p_{a=0,b=1}\}$	{0.95}			{0.90}			{0.86}		
R^2	3.4%	4.4%	2.9%	4.8%	3.0%	2.9%	62.3%	69.4%	0.9%
$R^2_{a=0,b=1}$	3.4%	3.7%	1.4%	4.6%	5.0%	2.7%	62.3%	62.6%	-22.0%
R^2_{OOS}		5.3%	5.7%		11.0%	11.4%		63.2%	-8.2%

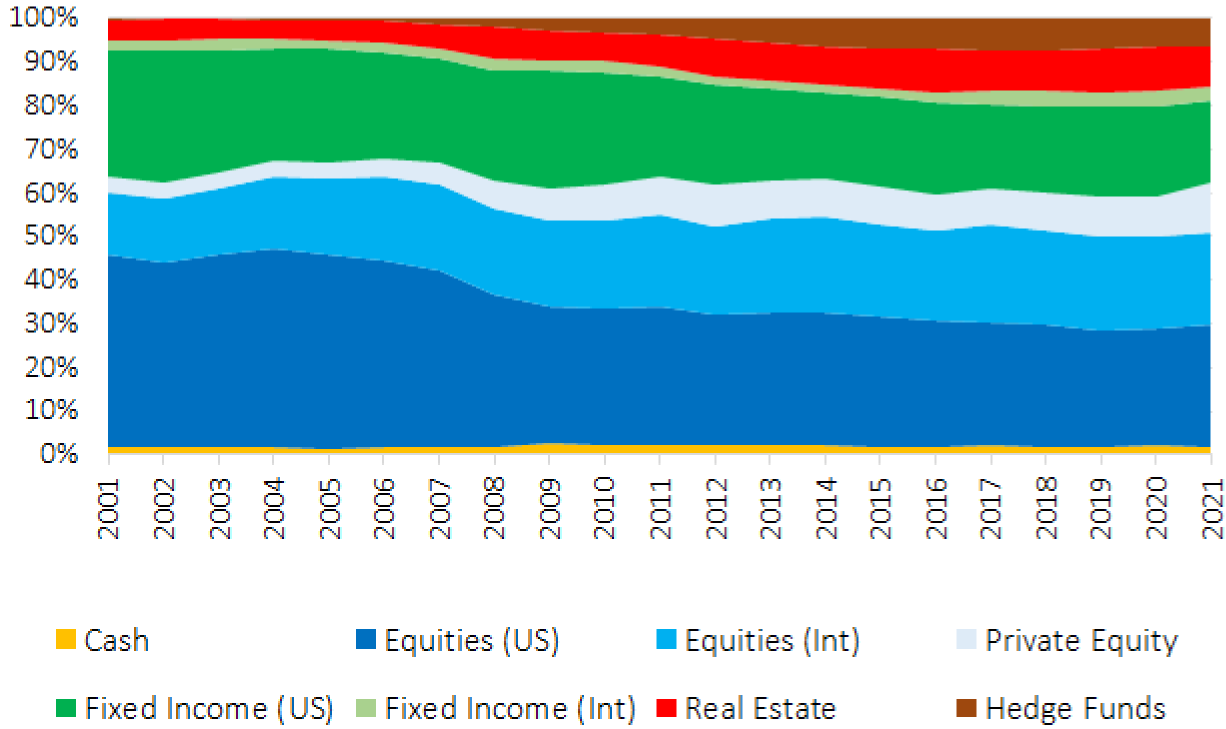


Figure IA.1
Aggregate Allocation of US Public Pension Funds

This figure plots the aggregate allocation of US Public Pension Funds from 2001 to 2021 (data from the Center for Retirement Research at Boston College). This allocation is used to form the market portfolio in the Pension CAPM ($m = p$) model we explore in the main text.

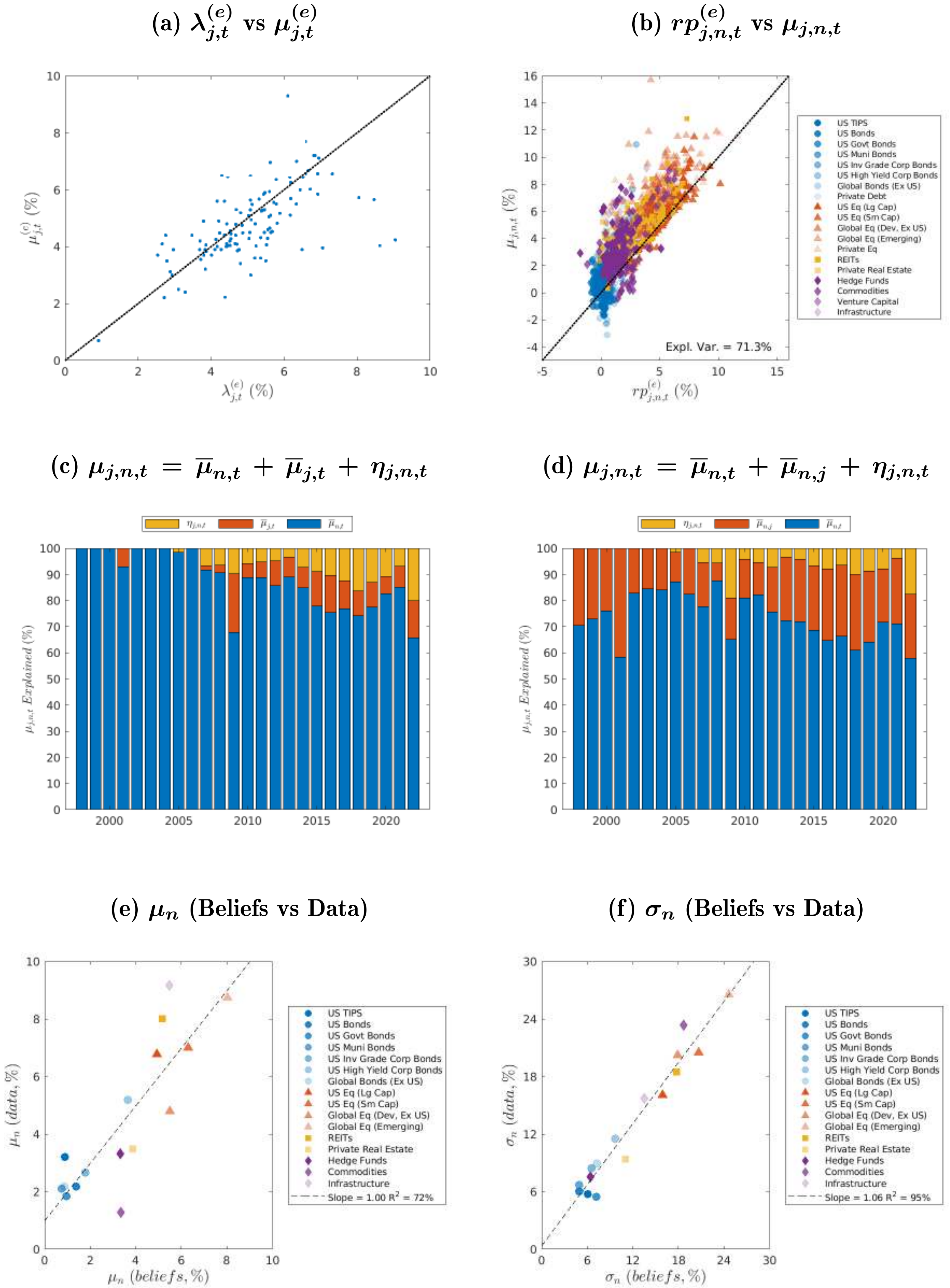


Figure IA.2

Main Results: Using Only Asset Managers

This figure replicates our main results, but using only the CMAs of asset managers. Panel (a) replicates Figure 3(a) in the main text. Panel (b) replicates Figure 5(a) in the main text. Panels (c) and (d) replicate Figures 7(a) and 7(b) in the main text. Panels (e) and (f) replicate Figures 11(a) and 11(b) in the main text. Sections 2 and A provide more details about our subjective beliefs data and Section B.1.1 provides more details about the analysis reported in this figure.

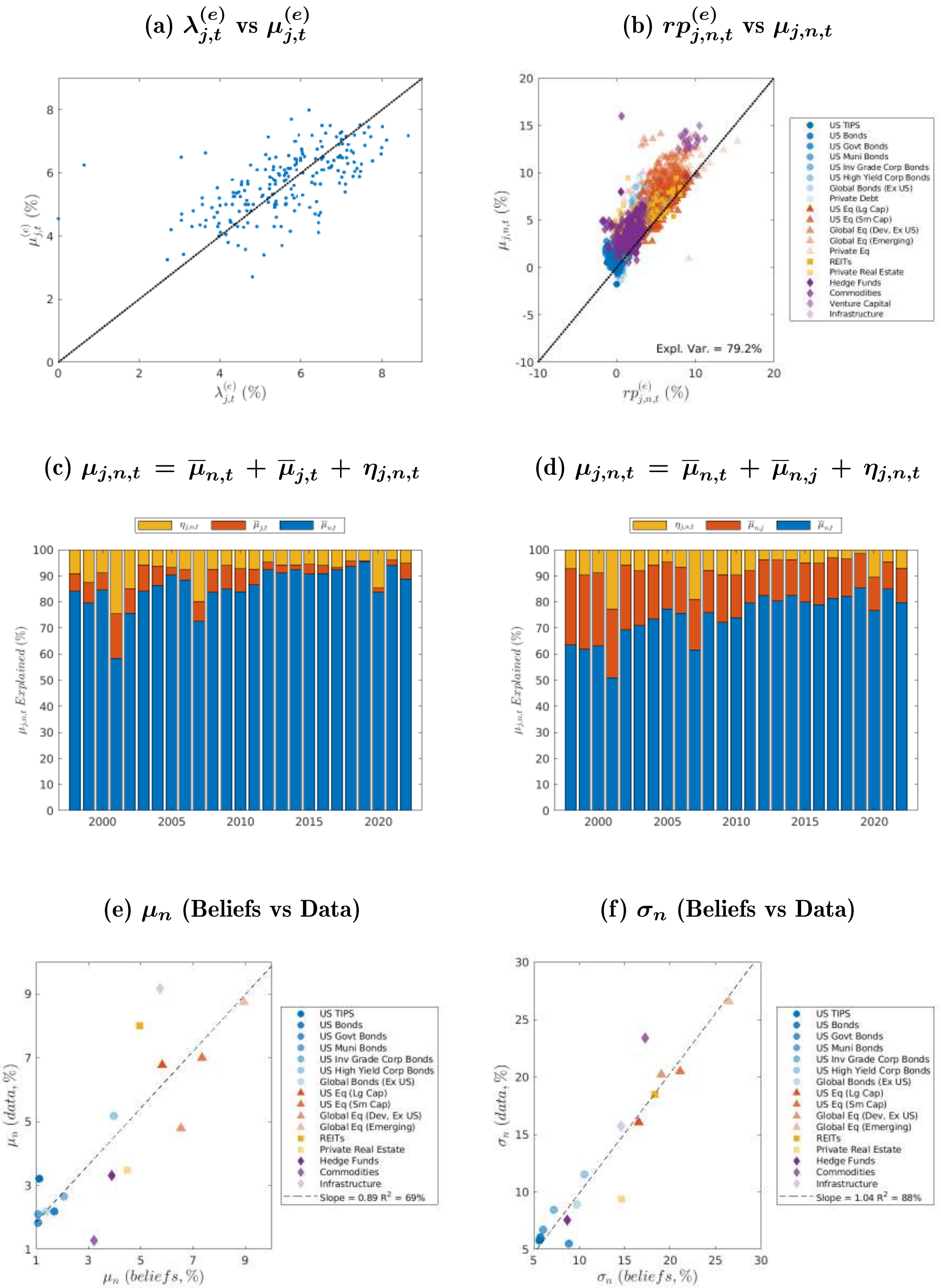


Figure IA.3

Main Results: Using Only Institutional Investor Consultants

This figure replicates our main results, but using only the CMAs of institutional investor consultants. Panel (a) replicates Figure 3(a) in the main text. Panel (b) replicates Figure 5(a) in the main text. Panels (c) and (d) replicate Figures 7(a) and 7(b) in the main text. Panels (e) and (f) replicate Figures 11(a) and 11(b) in the main text. Sections 2 and A provide more details about our subjective beliefs data and Section B.1.1 provides more details about the analysis reported in this figure.

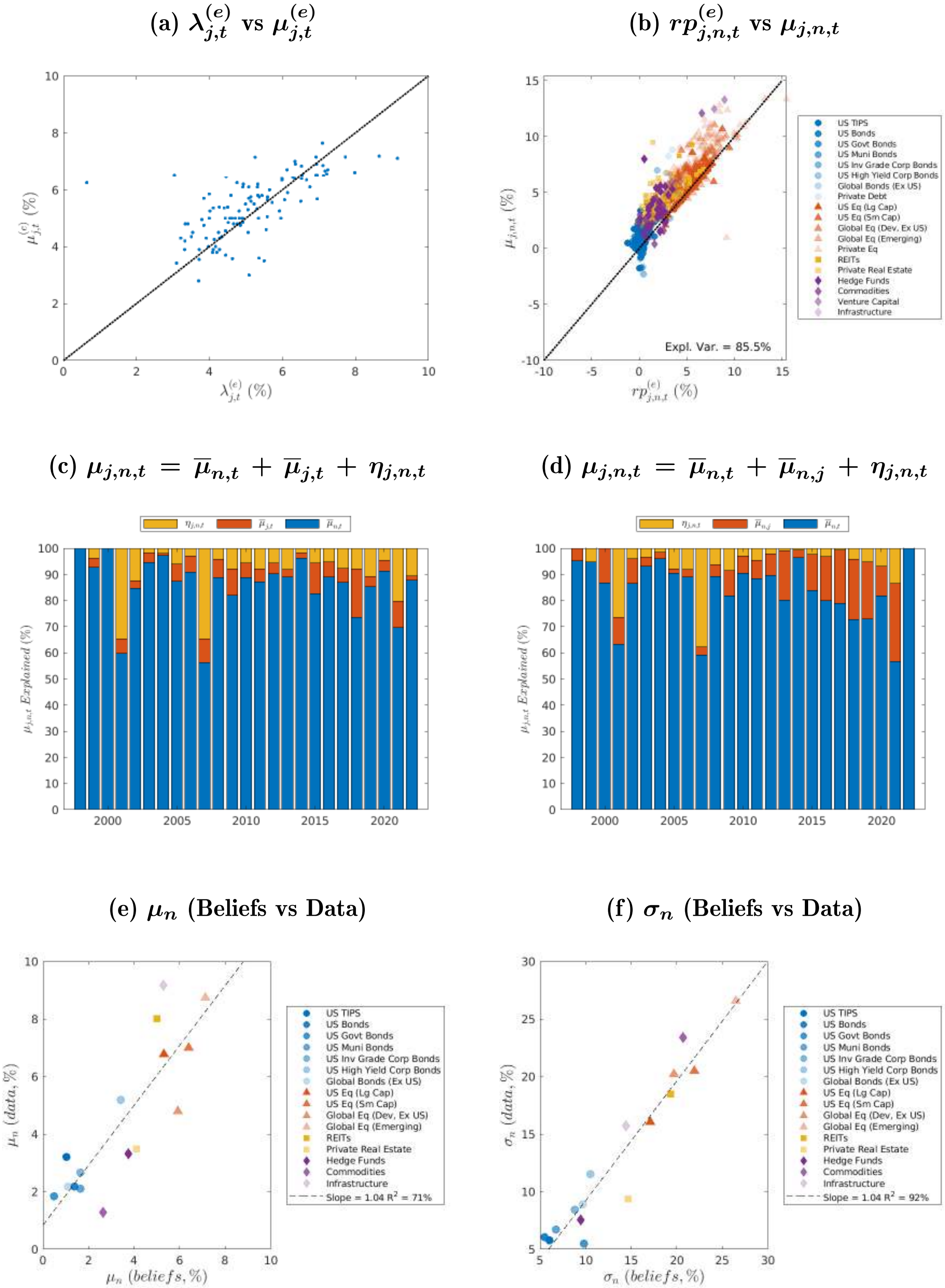


Figure IA.4

Main Results: Using Only Data Obtained Indirectly through Pension Fund Reports

This figure replicates our main results, but using only data obtained indirectly through pension fund reports. Panel (a) replicates Figure 3(a) in the main text. Panel (b) replicates Figure 5(a) in the main text. Panels (c) and (d) replicate Figures 7(a) and 7(b) in the main text. Panels (e) and (f) replicate Figures 11(a) and 11(b) in the main text. Sections 2 and A provide more details about our subjective beliefs data and Section B.1.2 provides more details about the analysis reported in this figure.

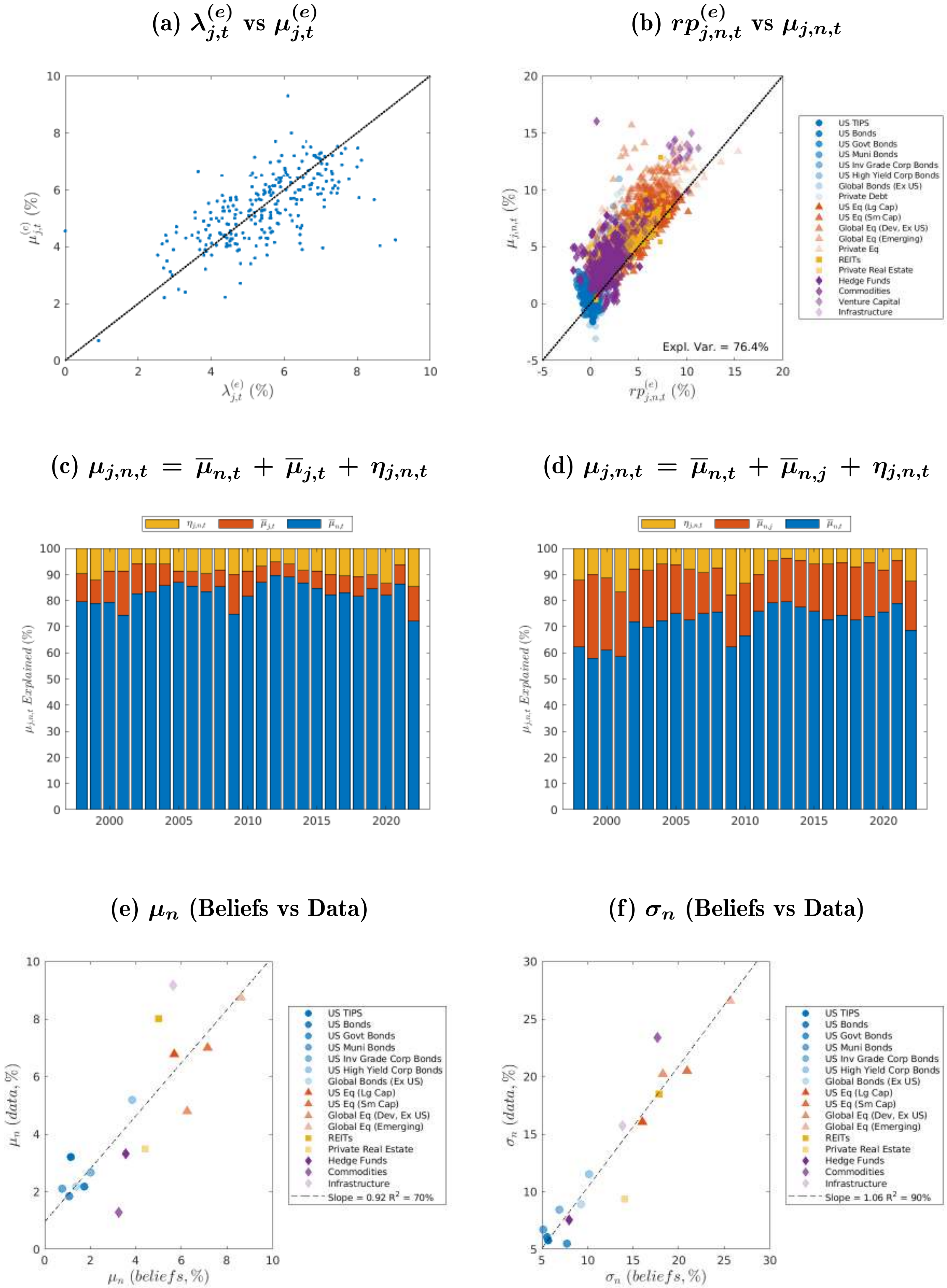


Figure IA.5

Main Results: Using Only Data Obtained Directly from the Institutions Underlying our CMAs

This figure replicates our main results, but using only data obtained directly from the Institutions Underlying our CMAs. Panel (a) replicates Figure 3(a) in the main text. Panel (b) replicates Figure 5(a) in the main text. Panels (c) and (d) replicate Figures 7(a) and 7(b) in the main text. Panels (e) and (f) replicate Figures 11(a) and 11(b) in the main text. Sections 2 and A provide more details about our subjective beliefs data and Section B.1.2 provides more details about the analysis reported in this figure.

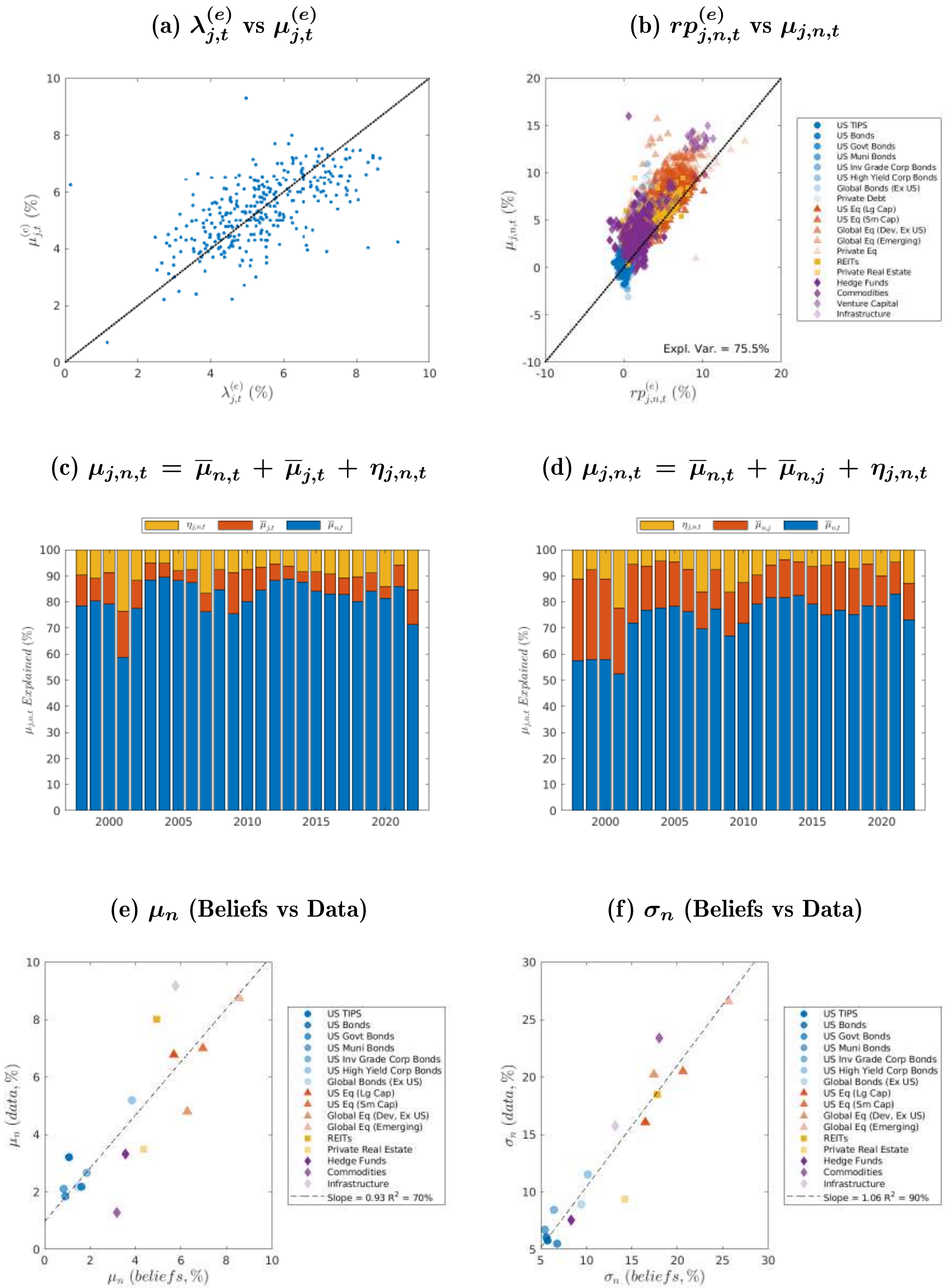


Figure IA.6

Main Results: Using Only Data with Primary Asset Class Available

This figure replicates our main results, but using only institution-year-asset class observations for which the given primary asset class used in the analysis is directly available among our master asset classes for the given institution-year observation. Panel (a) replicates Figure 3(a) in the main text. Panel (b) replicates Figure 5(a) in the main text. Panels (c) and (d) replicate Figures 7(a) and 7(b) in the main text. Panels (e) and (f) replicate Figures 11(a) and 11(b) in the main text. Sections 2 and A provide more details about our subjective beliefs data and Section B.1.3 provides more details about the analysis reported in this figure.

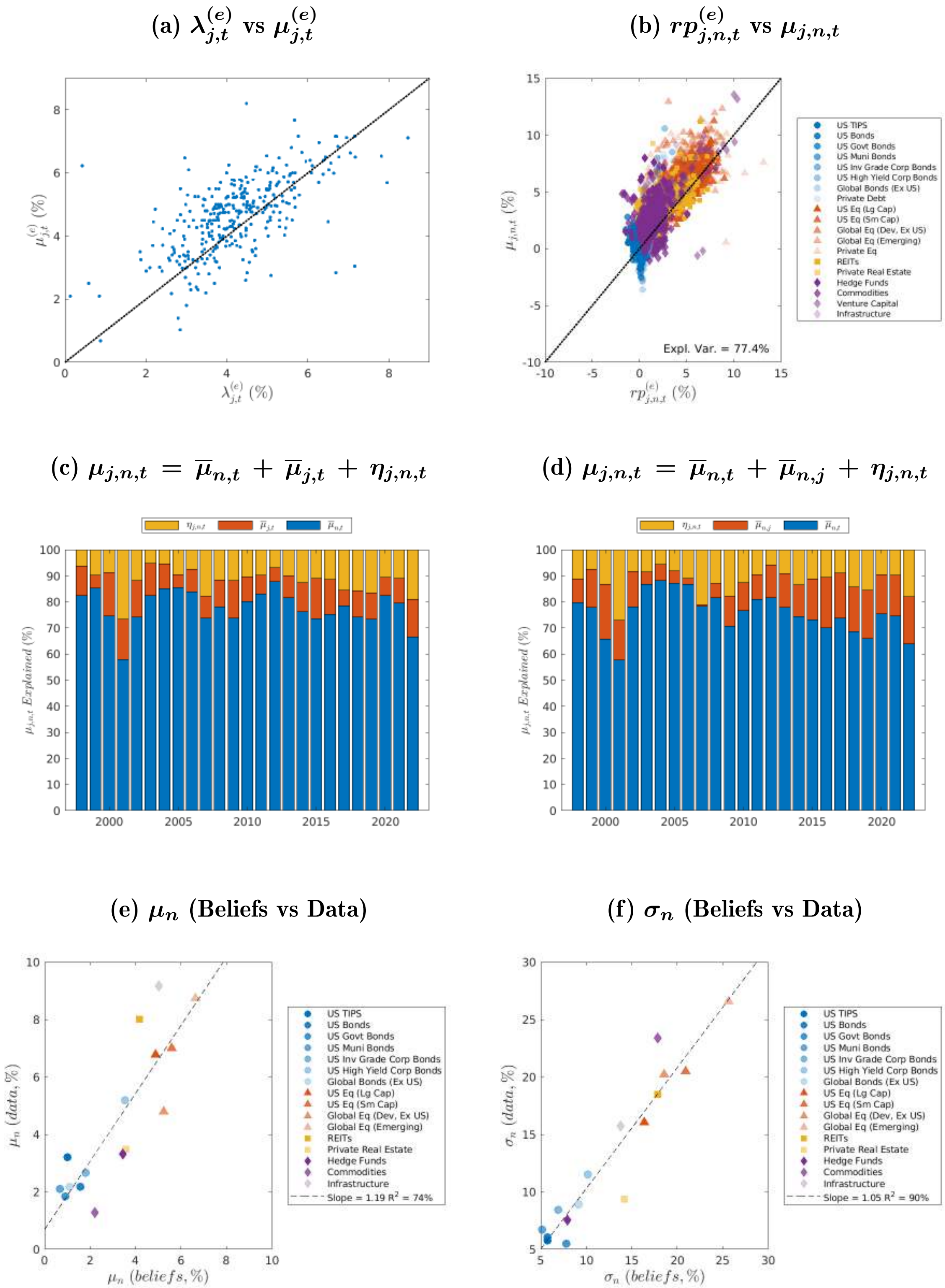


Figure IA.7

Main Results: $\mathbb{E}[R]$ Based on Expected Geometric Returns

This figure replicates our main results, but measuring $\mathbb{E}[R]$ from expected geometric returns (i.e., using $\mathbb{E}_{j,t}[R] = e^{\widehat{\mu}_{j,t}} - 1$, where $\widehat{\mu}_{j,t}$ is the expected log return discussed in Subsection A.2). Panel (a) replicates Figure 3(a) in the main text. Panel (b) replicates Figure 5(a) in the main text. Panels (c) and (d) replicate Figures 7(a) and 7(b) in the main text. Panels (e) and (f) replicate Figures 11(a) and 11(b) in the main text. Sections 2 and A provide more details about our subjective beliefs data and Section B.2.1 provides more details about the analysis reported in this figure.

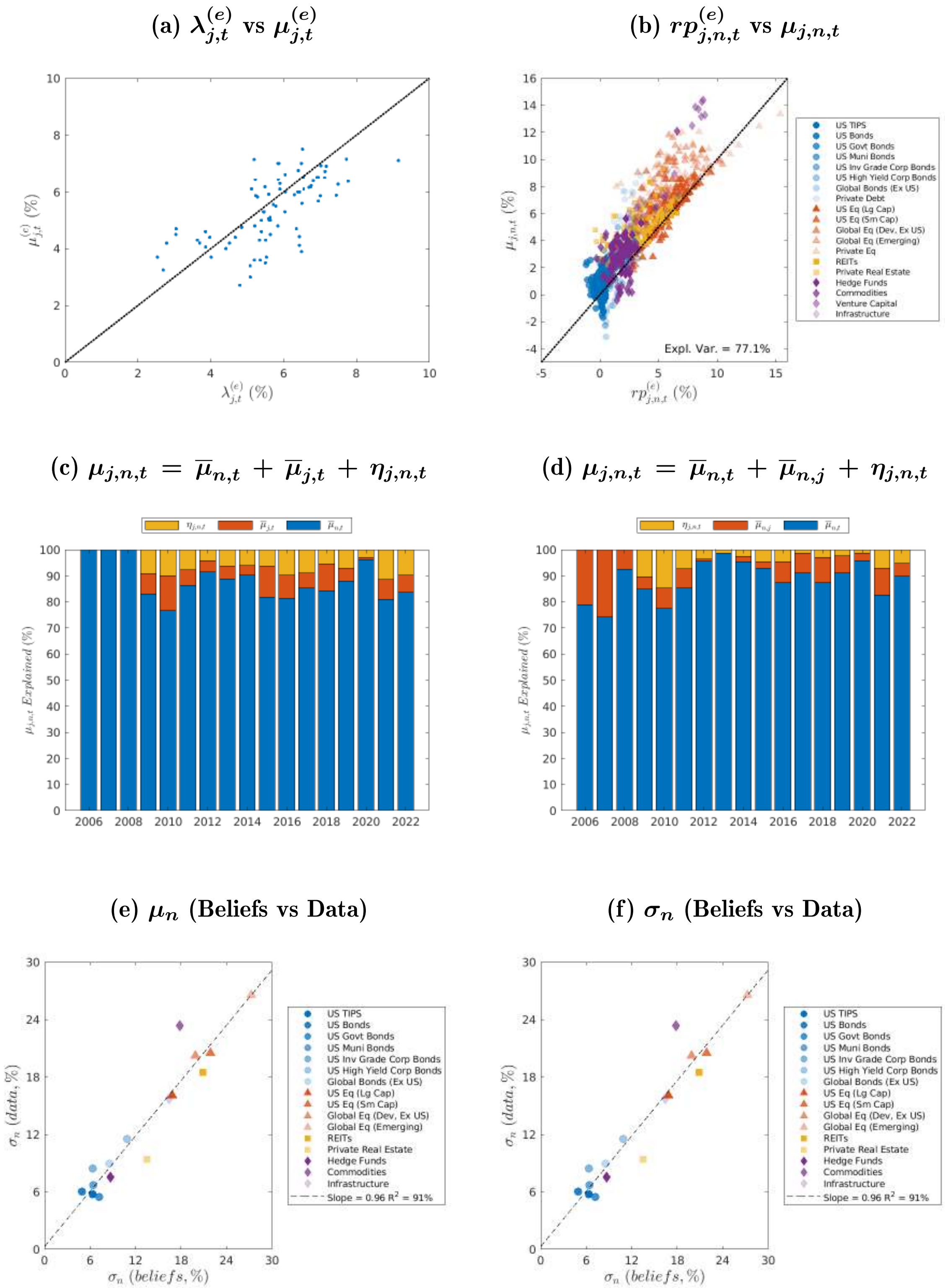


Figure IA.8
Main Results: Beliefs Based on a Homogeneous Horizon

This figure replicates our main results, but measuring beliefs only from CMAs that state a 10 year horizon, which is the most common horizon in our dataset. Panel (a) replicates Figure 3(a) in the main text. Panel (b) replicates Figure 5(a) in the main text. Panels (c) and (d) replicate Figures 7(a) and 7(b) in the main text. Panels (e) and (f) replicate Figures 11(a) and 11(b) in the main text. Sections 2 and A provide more details about our subjective beliefs data and Section B.2.2 provides more details about the analysis reported in this figure.

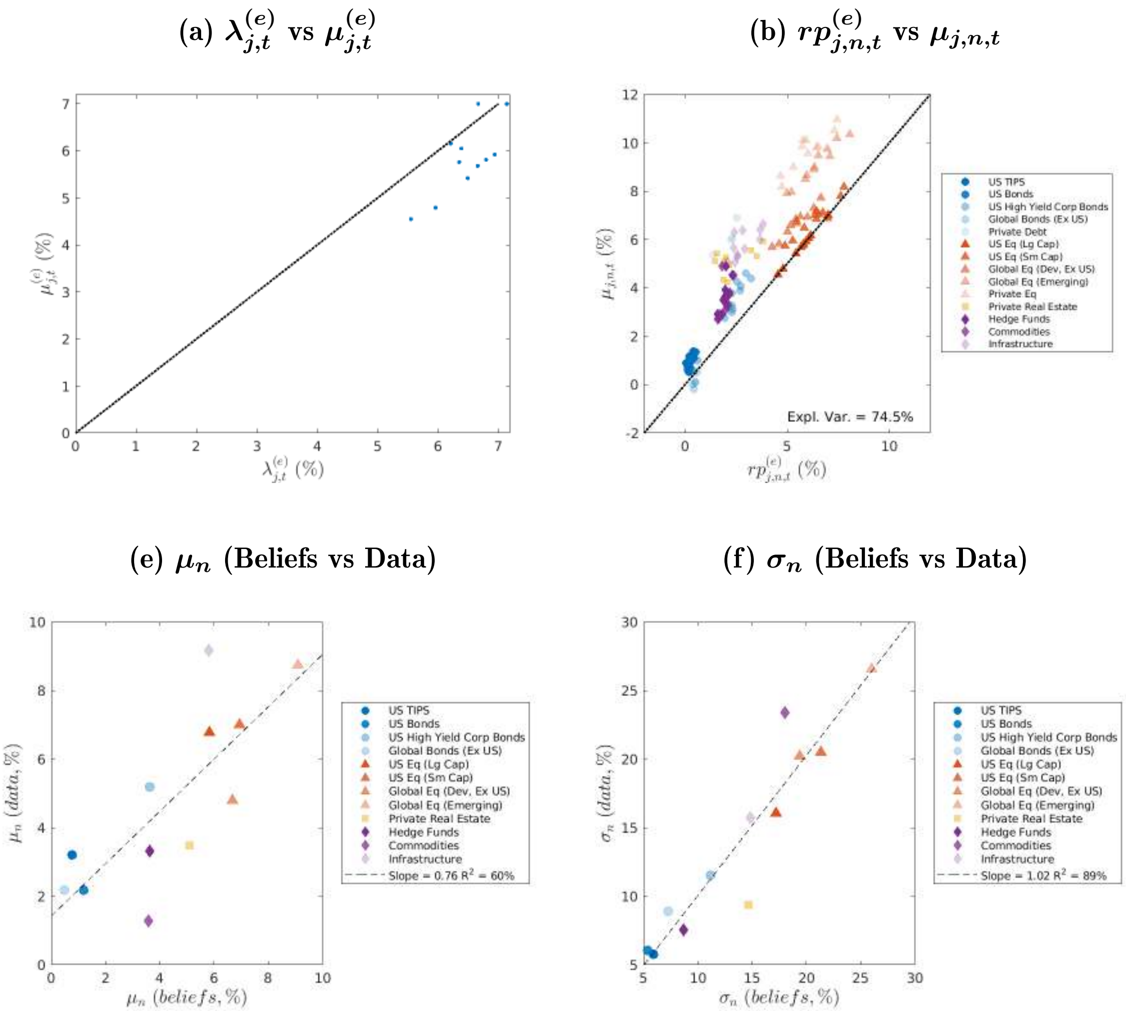


Figure IA.9

Main Results: Using Aggregated Data Obtained from a Third Party Company

This figure replicates our main results, but measuring beliefs each year from CMAs collected from a third party company and aggregated across institutions (we only have access to the aggregated data). Given this aggregation issue, we cannot replicate our results about heterogeneity of beliefs across institutions in this sample. Panel (a) replicates Figure 3(a) in the main text. Panel (b) replicates Figure 5(a) in the main text. Panels (c) and (d) replicate Figures 11(a) and 11(b) in the main text. Sections 2 and A provide more details about our subjective beliefs data and Section B.3 provides more details about the analysis reported in this figure.